What to do today (Feb 8)?

1. Introdution and Preparation

2. Analysis with Binary Variables (Chp1-2)

- *2.1 Analysis with binary variables I (Chp 1)*
- *2.2 Analysis with binary response (Chp 2)*
  - *2.2.1 Regression models (Chp2.1, Chp2.2.1)*
  - *2.2.2 Simple logistic regression analysis (Chp2.2.2-7)*
  - *2.2.3 Multiple logistic regression analysis (Chp2.2.2-7)*
- **2.3 Generalized linear models (Chp2.3)**

3. Analysis with Multicategory Variables (Chp3)

- **3.1 Analysis of larger contingency tables**
- *3.2 Regression analysis with multicategory response*

2.3A Generalized linear models: Introduction

Recall that a regression model describes patterns of association and interaction, and can be used to predict "future": **very useful**

$\Longrightarrow$ **Is there a broad class of regression models includes OLM, Logit models as special cases?**

Generalized Linear Models (GLM): a unified framework for many regression analyses.

- including OLM, Logit models as special cases
- including other regression models

2.3B Generalized linear models: Components

What common features in the examples of regression models, OLM, Logit? Recall how to conduct the analysis with *R: glm(formula, family)*
GOAL: to study how $Y \leftarrow X_1, \ldots, X_K$

Generalized Linear Models:

- **Random Component.** response r.v. Y with $\mu(x_1, \ldots, x_k) = E(Y|x_1, \ldots, x_k)$ to be examined
- **Systematic Component.** $\alpha + \beta_1 x_1 + \ldots + \beta_K x_K$
  Some x_k can be based on others: e.g. $x_3 = x_1 x_2$.
- **Link Function.** $g(\mu) = \alpha + \beta_1 x_1 + \ldots + \beta_K x_K$
  The link function $g(\cdot)$ links the *random componet* through its mean and the *systematic component*. **More on GLM later**

3.1.1 Review of two-way contingency tables

For general categorical variables X and Y, ...
$I \times J$ **contingency table**: cell counts n_{ij}, $i = 1, \ldots, I$ and $j = 1, \ldots, J$

Cell Counts

	Variable Y				
Variable X	1	2	...	J	total
1	n_{11}	n_{12}	...	n_{1J}	n_{1+}
2	n_{21}	...	...	n_{2J}	n_{2+}
...	...	...	...	...	
I	n_{I1}	...	...	n_{IJ}	n_{I+}
total	n_{+1}	...	...	n_{+J}	n_{++}

- subtotals: n_{i+} (row totals); n_{+j} (column totals)
- grand total: n_{++}

How to analyze the 2-Way contingency table?

3.1.1 Review of two-way contingency tables: Joint, Marginal and Conditional Probabilities

Consider two discrete r.v.s X and Y, with all possible levels $i = 1, \ldots, I$ for X and $j = 1, \ldots, J$ for Y

- **joint distn (prob) of X and Y:** $\pi_{ij} = Pr(X = i, Y = j)$ for $i = 1, \ldots, I$ and $j = 1, \ldots, J$
- **marginal distn (prob) of X,Y:** $\pi_{i+} = Pr(X = i)\ \ i = 1, \ldots, I$; $\pi_{+j} = Pr(Y = j)\ \ j = 1, \ldots, J$

  $\pi_{i+} = \sum_{j=1}^{J} \pi_{ij}$ and $\pi_{+j} = \sum_{i=1}^{I} \pi_{ij}$
- **conditional disn (prob) of $X|Y$ or $Y|X$:**
  $Pr(X = i|Y = j) = \frac{\pi_{ij}}{\pi_{+j}} = \pi_{i|j}$, $\ Pr(Y = j|X = i) = \frac{\pi_{ij}}{\pi_{i+}} = \pi_{j|i}$ for $i = 1, \ldots, I$ and $j = 1, \ldots, J$

3.1.1 Review of two-way contingency tables: Joint, Marginal and Conditional Probabilities

Probabilities with $I \times J$ contingency table: π_{ij}, $i = 1, \ldots, I$ and $j = 1, \ldots, J$

Probabilities in $I \times J$ Table

	Variable Y				
Variable X	1	2	...	J	total
1	π_{11}	π_{12}	...	π_{1J}	π_{1+}
2	π_{21}	...	...	π_{2J}	π_{2+}
...	...	...	...	...	
I	π_{I1}	...	...	π_{IJ}	π_{I+}
total	π_{+1}	...	...	π_{+J}	$\pi_{..} \equiv 1$

- marginal probabilitites: π_{i+} (with the row variable X); π_{+j} (with the column variable Y)
- conditional probabilitites: $\pi_{ij}/\pi_{i+} = \pi_{j|i}$ (conditional on X=i); $\pi_{ij}/\pi_{+j} = \pi_{i|j}$ (conditional on Y=j)

3.1.1 Review of two-way contingency tables: $X \perp\!\!\!\perp Y$

independence: $X \perp\!\!\!\perp Y$ (*the simplest relationship of* X *and* Y)

- $X \perp\!\!\!\perp Y$ when $\pi_{ij} = \pi_{i+}\pi_{+j}$ for all $i = 1, \ldots, I$ and $j = 1, \ldots, J$, or
- $X \perp\!\!\!\perp Y$ when $P(X = i|Y = j) = \pi_{i|j}$ is π_{i+} for all i, j, or $P(Y = j|X = i) = \pi_{j|i}$ is π_{+j} for all i, j
- When X and Y are binary, $X \perp\!\!\!\perp Y$ if $OR = 1$.

3.1.1 Review of two-way contingency tables: Probability Models

- Consider r.v.s X with I levels and Y with J levels
- Consider observations of (X, Y): $(X_1, Y_1), \ldots, (X_n, Y_n)$

Data are summarized by the $I \times J$ contingency table: cell counts N_{ij}, $i = 1, \ldots, I$ and $j = 1, \ldots, J$

Cell Counts

	Variable Y				
Variable X	1	2	...	J	total
1	N_{11}	N_{12}	...	N_{1J}	N_{1+}
2	N_{21}	...	...	N_{2J}	N_{2+}
...	...	...	...	...	...
I	N_{I1}	...	...	N_{IJ}	N_{I+}
total	N_{+1}	...	...	N_{+J}	N_{++}

- subtotals: N_{i+} (row totals); N_{+j} (column totals)
- grand total: N_{++}

3.1.1 Review of two-way contingency tables: Probability Models

- **multinomial sampling:**
  With fixed $N_{++} = n$,
  $(N_{ij} : i = 1, \ldots, I; j = 1, \ldots, J) \sim \mathit{multinomial}(n, \pi'_{ij}s)$

- **purposive sampling:**
  Given $N_{i+} = n_{i+}$,
  $(N_{ij} : j = 1, \ldots, J) \sim \mathit{multinomial}(n_{i+}, \pi'_{j|i}s)$ with
  $\pi_{j|i} = \pi_{ij}/\pi_{i+}$ and $\sum_{j=1}^{J} \pi_{j|i} = 1$.

3.1.2 Analysis of $I \times J$ contingency table: Estimating Probabilities

- **To estm** π_{ij} with data from cross-sectional studies by multinomial sampling:

Given the grand total $N_{++} = n$,
$(N_{ij} : i = 1, \ldots, I; j = 1, \ldots, J) \sim \textit{multinomial}(n, \pi_{ij}'s)$

- the likelihood function (with constraint $\sum \pi_{ij} = 1$):

$$L(\pi_{ij}'s|data) \propto \prod_{i=1,\ldots,I;j=1,\ldots,J} \pi_{ij}^{n_{ij}}$$

$\Longrightarrow$ the MLE $\hat{\pi}_{ij} = n_{ij}/n$, for $i = 1, \ldots, I; j = 1, \ldots, J$

the same as the corresponding sample proportions!

3.1.2 Analysis of $I \times J$ contingency table: Estimating Probabilities

Plus, $\hat{\pi}_{1+} = \hat{\pi}_{11} + \ldots + \hat{\pi}_{1J} = n_{1+}/n$, ...,
and $\hat{\pi}_{j|1} = \hat{\pi}_{1j}/\hat{\pi}_{1+} = n_{1j}/n_{1+}$, ...

the same as the corresponding sample proportions!

$\Longrightarrow$ confidence intervals: Wald type, score based, LRT based with large sample

e.g. Wald type: $\hat{\pi}_{11} \pm (1.96)\sqrt{\frac{\hat{\pi}_{11}[1-\hat{\pi}_{11}]}{n}}$

3.1.2 Analysis of $I \times J$ contingency table: Estimating Probabilities

How about the estimation with data collected by purposive sample with n_{i+} fixed by study design?
Is there enough information for estimating π_{ij} with the data?

- **To estm** $\pi_{1|i} = \pi_{i1}/\pi_{i.}, \ldots, \pi_{J|i} = \pi_{iJ}/\pi_{i.}$ with data from case-control studies by purposive sampling given row totals $n_{i.}$'s, $i = 1, \ldots, I$: $\sum_{j=1}^{J} \pi_{j|i} = 1$

the likelihood functions:

$$L(\pi_{j|1} : j = 1, \ldots, J | \textit{data in line } 1) \propto \prod_{j=1}^{J} \pi_{j|1}^{n_{1j}}$$

... ...

$$L(\pi_{j|I} : j = 1, \ldots, J | \textit{data in line } I) \propto \prod_{j=1}^{J} \pi_{j|I}^{n_{Ij}}$$

$\Longrightarrow$ the MLE $\hat{\pi}_{j|1} = n_{1j}/n_{1.}$, $\hat{\pi}_{j|2} = n_{2j}/n_{2.}$, ..., and $\hat{\pi}_{j|I} = n_{Ij}/n_{I.}$ for $j = 1, \ldots, J$.

$\Longrightarrow$ confidence intervals: Wald type, score based, LRT based with large sample
e.g. Wald type CI: $\hat{\pi}_{j|1} \pm 1.96\sqrt{\hat{\pi}_{j|1}[1-\hat{\pi}_{j|1}]/n_{1.}}$ for $j = 1, \ldots, J$.

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

In general, testing on H_0 vs H_1 with an $I \times J$ contingency table:

Cell Counts

	Variable Y				
Variable X	1	2	...	J	total
1	n_{11}	n_{12}	...	n_{1J}	n_{1+}
2	n_{21}	...	...	n_{2J}	n_{2+}
...	...	...	...	...	
I	n_{I1}	...	...	n_{IJ}	n_{I+}
total	n_{+1}	...	...	n_{+J}	n_{++}

Suppose, when H_0 is true, the expected frequencies $E_{H_0}(N_{ij}) = \mu_{ij}$.
Then test on H_0 by comparing N_{ij} with μ_{ij}?

Pearson's Chi-Squared Test (K. Pearson, 1900) on H_0 vs H_1 with an $I \times J$ contingency table
Consider the **Pearson χ^2-statistic**:

$$\mathcal{X}^2 = \sum_{i,j} \frac{(N_{ij} - \mu_{ij})^2}{\mu_{ij}}$$

Approximately, $\mathcal{X}^2 \sim \chi^2(df)$ under H_0 for large n.
$\Longrightarrow p - value = P_{H_0}(\mathcal{X}^2 \geq \mathcal{X}^2_{obs})$

Remarks:

- The χ^2-approximation is good usually when $\mu_{ij} \geq 5$
- The degrees of freedom:
  $df = \#$(parameters under H_1) - $\#$(parameters under H_0)
- What are μ_{ij}? How to implement the procedure?

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

Likelihood Ratio Test (LRT) on H_0 vs H_1 with an $I \times J$ contingency table

Consider the **likelihood ratio test statistic**:

$$-2\log\left(\frac{\max L_{H_0}(parameter|data)}{\max L(parameter|data)}\right) \propto G^2 = 2\sum_{i,j} N_{ij}\log\left(\frac{N_{ij}}{\mu_{ij}}\right)$$

Approximately, $G^2 \sim \chi^2(df)$ under H_0 for large n

$\Longrightarrow p - value = P_{H_0}(X^2 \geq X^2_{obs})$

- the χ^2-approximation is good usually when $\mu_{ij} \geq 5$
- the degrees of freedom
  df = #(parameters under H_1) - #(parameters under H_0)
- What are μ_{ij}? How to implement the procedure?

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

Consider specifically ...

To test on $H_0 : X \perp\!\!\!\perp Y$ vs $H_1 : X \not\perp\!\!\!\perp Y$ with an $I \times J$ contingency table by the multinomial sampling with $N_{++} = n$

- **Reformulate the hypotheses according to the sampling ...**
  $H_0 : \pi_{ij} = \pi_{i+}\pi_{+j}$ for all i, j vs H_1 : otherwise

- **Getting μ_{ij} or their best estimates ... ...**
  $\mu_{ij} = E_{H_0}(N_{ij}) = n\pi_{i+}\pi_{+j}$
  the MLE $\hat{\mu}_{ij} = n\hat{\pi}_{i+}\hat{\pi}_{+j} = \frac{n_{i+}n_{+j}}{n}$

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

- **Applying Pearson's χ^2-test ...**
  - determine $df = (IJ - 1) - ([I-1] + [J-1]) = (I-1)(J-1)$ by Fisher (1922)
  - calculate $\mathcal{X}^2_{obs} = \sum \frac{(n_{ij} - \hat{\mu}_{ij})^2}{\hat{\mu}_{ij}}$
  - calculate $p - value = P_{H_0}(\mathcal{X}^2 \geq \mathcal{X}^2_{obs})$ based on $\mathcal{X}^2 \sim \chi^2(df)$ approximately when $n >> 1$
  - draw conclusion

- **Applying LRT-test ...**
  - determe $df = (IJ - 1) - ([I-1] + [J-1]) = (I-1)(J-1)$
  - calculate $G^2_{obs} = 2 \sum_{i,j} n_{ij} \log \left(\frac{n_{ij}}{\hat{\mu}_{ij}} \right)$
  - calculate $p - value = P_{H_0}(G^2 \geq G^2_{obs})$
  - draw conclusion

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

To test on $H_0 : X \perp\!\!\!\perp Y$ vs $H_1 : X \not\perp\!\!\!\perp Y$ with an $I \times J$ contingency table by the purposive sampling with $N_{i+} = n_{i+}$

- **Reformulate the hypotheses according to the sampling ...**
  $H_0 : \pi_{j|i} = \pi_{+j}$ for all i, j vs H_1 : otherwise
  $(\pi_{j|i} = \frac{\pi_{ij}}{\pi_{i+}} = P(Y = j|X = i))$
- **Getting μ_{ij} or their best estimates ... ...**
  $\mu_{ij} = E_{H_0}(N_{ij}) = n_{i+}\pi_{j|i}$
  the MLE $\hat{\mu}_{ij} = n_{i+}\hat{\pi}_{j|i} = n_{i+}\frac{n_{+j}}{n} = \frac{n_{i+}n_{+j}}{n}$

3.1.2 Analysis of $I \times J$ contingency table: Hypothesis Testing on Independence

- **Applying Pearson's χ^2-test ...**
  - determe $df = (IJ - I) - (J - 1) = (I - 1)(J - 1)$
  - calculate $\mathcal{X}^2_{obs} = \sum \frac{(n_{ij} - \hat{\mu}_{ij})^2}{\hat{\mu}_{ij}}$
  - calculate $p - value = P_{H_0}(\mathcal{X}^2 \geq \mathcal{X}^2_{obs})$ based on $\mathcal{X}^2 \sim \chi^2(df)$ approximately when $n >> 1$
  - concluding
- **Applying LRT ...**
  - determe $df = (IJ - I) - (J - 1) = (I - 1)(J - 1)$
  - calculate $G^2_{obs} = 2 \sum_{i,j} n_{ij} \log \left(\frac{n_{ij}}{\hat{\mu}_{ij}} \right)$
  - calculate $p - value = P_{H_0}(G^2 \geq G^2_{obs})$
  - concluding

The same test statistics $\mathcal{X}^2$, G^2 as used with tables by the multinominal sampling.

3.1.3A Multi-Way Contingency Table: Introduction

What if three categorical variables X (with I levels), Y (with J levels), and Z (with K levels) are of interest?

$\Longrightarrow$ studying about three-way contingency tables, $I \times J \times K$ tables: the statistical analyses with them.

What can be new in the relevant statistical analysis, compared to analysis with two-way contingency tables?

Can all the relevant goals be achieved by studying the pairs, (X, Y), (X, Z) and (Y, Z)?

Let's see an example ... ...

3.1.3A Multi-Way Contingency Table: Introduction

Example. Death Penalty (e.g. Agresti, 2006) A $2 \times 2 \times 2$ contingency table from an article on effects of racial characteristics on whether individuals convicted of homicide receive the death penalty.
X =Defendant's Race: black vs white; Y =Death Penalty: yes vs not;
Z =Victim's Race: black vs white

Victims' Race	Defendant's Race	Death Penalty Yes	Death Penalty No
white	white	53	414
	black	11	37
black	white	0	16
	black	4	139

3.1.3A Multi-Way Contingency Table: Introduction

Analysis 1. Partial tables: X-Y at Z=white, black

Victims' Race	Defendant's Race	Death Penalty Yes	Death Penalty No	Percentage (row) Yes
white	white	53	414	11.3
	black	11	37	22.9
black	white	0	16	0.0
	black	4	139	2.8

- Table X-Y at Z=white: Death penalty imposed to black is 22.9% - 11.3%=11.6% higher than to white for white victim
- Table X-Y at Z=black: Death penalty imposed to black is 2.8% - 0%=2.8% higher than to white for black victim

3.1.3A Multi-Way Contingency Table: Introduction

Analysis 2. X-Y marginal table:

Defendant's Race	Death Penalty Yes	Death Penalty No	percentage (row) Yes
white	53=(53+0)	430=(414+16)	11.0
black	15=(11+4)	176=(37+139)	7.9

Death penalty imposed to white is 11.0%-7.9% = 3.1% higher than to black overall (ignoring victim's race)

Sample odds ratio = 1.415

3.1.3A Multi-Way Contingency Table: Introduction

Why the association bt death penalty verdict and defendant's race differ so much when we ignore vs control victims' race?

The marginal association can have different direction from the conditional associations: **Simpson's paradox**

Has it something to do with the nature of the association bt the control variable, victims' race (Z), and each of the other variables (X and Y)?

3.1.3A Multi-Way Contingency Table: Introduction

Analysis 3. X-Z marginal table:

Defendant's Race	Victims' Race	
	white	black
white	467	48
black	16	143

Sample odds ratio = 87.0 (the odds that a white defendant had white victims is estimated to be 87.0 times the odds that a black defendant had white victims)

This explains why the conclusions by Analyses 1. and 2. aren't consistent.

3.1.3A Multi-Way Contingency Table: Introduction

Analysis 4. Y-Z marginal table:

Victims' Race	Death Penalty Yes	Death Penalty No	percentage (row) Yes
white	64	451	14.2
black	4	155	2.6

Sample odds ratio = 5.50 (the odds that death penalty to a defendant who had white victims is estimated to be 5.50 times the odds that death penalty to a defendant who had black victims)

Probably this should be the key finding.

3.1.3B Multi-Way Contingency Table: Basic Concepts

- Two-Way **Partial Table:** any two-way cross-sectional slice all the cell counts associated with two of the three variables and with the 3rd variable fixed at a level
  e.g. $\{n_{ij2} : i = 1, \ldots, I; j = 1, \ldots, J\}$
  $\Longrightarrow$ telling only about X and Y $|Z = 2$, a conditional association

- Two-Way **Marginal Table:** any two-way table obtained by combining the two-way partial tables according to the 3rd variable
  e.g. $\{n_{ij+} : i = 1, \ldots, I; j = 1, \ldots, J\}$
  $\Longrightarrow$ telling only about X and Y accross diff levels of Z, a marginal association

3.1.3B Multi-Way Contingency Table: Basic Concepts

With a three-way contingency table, say, an $I \times J \times K$ table: $\{n_{ijk} : i = 1, \ldots, I; j = 1, \ldots, J; k = 1, \ldots, K\}$

- **joint prob.** $\pi_{ijk} = P(X = i, Y = j, Z = k)$
- **marginal prob.** $P(X = i, Y = j) = \pi_{ij+}$, $P(X = i) = \pi_{i++}$, etc
- **conditional prob.**
  $P(X = i, Y = j | Z = k) = \pi_{ijk}/\pi_{++k} = \pi_{ij|k}$,
  $P(X = i | Y = j, Z = k) = \pi_{ijk}/\pi_{+jk} = \pi_{i|jk}$, etc

3.1.3B Multi-Way Contingency Table: Basic Concepts

When the sampling is by the cross-sectional sampling with fixed overall total $n_{+++} = n$

- the MLE $\hat{\pi}_{ijk} = n_{ijk}/n$
- the MLE $\hat{\pi}_{ij+} = n_{ij+}/n$, $\hat{\pi}_{i++} = n_{i++}/n$, etc
- the MLE $\hat{\pi}_{ij|k} = n_{ijk}/n_{++k}$, $\hat{\pi}_{i|jk} = n_{ijk}/n_{+jk}$, etc

When the sampling is by the purposive sampling with fixed subtotals, say, n_{++k}

- the MLE $\hat{\pi}_{ij|k} = n_{ijk}/n_{++k}$, $\hat{\pi}_{i|jk} = n_{ijk}/n_{+jk}$, etc

What are new in stats analysis with three-way tables?

3.1.3C Multi-Way Contingency Table: Conditional vs Marginal Associations

Consider a $2 \times 2 \times K$ three-way contingency table:
$\{n_{ijk} : i = 1, 2; j = 1, 2; k = 1, \ldots, K\}$ as the observed frequencies
$\pi_{ijk} = P(X = i, Y = j, Z = k)$, the joint prob;
$\mu_{ijk} = E(N_{ijk})$, the expected frequencies;

- **X-Y conditional odds ratios**: [describe conditional X-Y association] For $k = 1, \ldots, K$,

$$\theta_{XY(k)} = \frac{\pi_{11k}\pi_{22k}}{\pi_{12k}\pi_{21k}} = \frac{\mu_{11k}\mu_{22k}}{\mu_{12k}\mu_{21k}}$$

  - **sample X-Y conditional odds ratios**: $\hat{\theta}_{XY(k)} = \frac{n_{11k}n_{22k}}{n_{12k}n_{21k}}$
  - If $\theta_{XY(k)} \equiv$ constant, $\Longrightarrow$ "homogeneous" conditional X-Y association

3.1.3C Multi-Way Contingency Table: Conditional vs Marginal Associations

Consider a $2 \times 2 \times K$ three-way contingency table:
$\{n_{ijk} : i = 1, 2; j = 1, 2; k = 1, \ldots, K\}$ as the observed frequencies
$\pi_{ijk} = P(X = i, Y = j, Z = k)$, the joint prob; $\mu_{ijk} = E(N_{ijk})$, the expected frequencies

- **X-Y marginal odds ratios**: [describe marginal X-Y association]

$$\theta_{XY} = \frac{\pi_{11+}\pi_{22+}}{\pi_{12+}\pi_{21+}} = \frac{\mu_{11+}\mu_{22+}}{\mu_{12+}\mu_{21+}}$$

  - **sample X-Y marginal odds ratios**: $\hat{\theta}_{XY} = \frac{n_{11+}n_{22+}}{n_{12+}n_{21+}}$
  - If $\theta_{XY(k)} \equiv$ constant [homogeneous conditional X-Y association], $\theta_{XY(k)} \equiv \theta_{XY}$???

    Why is the Simpson's Paradox!

3.1.3C Multi-Way Contingency Table: Conditional vs Marginal Associations

Marginal vs Conditional Independence

- $X \perp\!\!\!\perp Y | Z \leftrightarrow$ (iff) $\theta_{XY(k)} = 1$ for all k
- $X \perp\!\!\!\perp Y \leftrightarrow$ (iff) $\theta_{XY} = 1$
- $X \perp\!\!\!\perp Y | Z \not\leftrightarrow X \perp\!\!\!\perp Y$

See an illustrative example ...

3.1.3C Multi-Way Contingency Table: Conditional vs Marginal Associations

Example. A hypothetical clinical trial with two arms, two sites, two types of outcome

		Y=Response	
Z=Clinic	X=Treatment	success	failure
1	A	18	12
	B	12	8
2	A	2	8
	B	8	32
	A	20	20
	B	20	40

- *Look 1.* $\hat{\theta}_{XY(1)} = \hat{\theta}_{XY(2)} = 1 \Longrightarrow X \perp\!\!\!\perp Y$ given a clinic [conditional homogeneous association X-Y]
- *Look 2.* $\hat{\theta}_{XY} = 2 \Longrightarrow X \not\perp\!\!\!\perp Y$ [Treatment A is over B]

3.1.3C Multi-Way Contingency Table: Conditional vs Marginal Associations

		Y=Response	
Z=Clinic	X=Treatment	success	failure
1	A	18	12
	B	12	8
2	A	2	8
	B	8	32

- *Look 3.* the clue?
  - $X - Z$: $\hat{\theta}_{XZ(s)} = \hat{\theta}_{XZ(f)} = 6 \Longrightarrow X \not\perp Z$: Clinic 1 had more with A; 2, B.
    [This should have been controled: randomized clinical trial.]
  - $Y - Z$: $\hat{\theta}_{YZ(A)} = \hat{\theta}_{YZ(B)} = 6 \Longrightarrow Y \not\perp Z$: clinic effect! Clinic 1 is over Clinic 2.
    [This should have been controled: the goal is to assess the treatment effect.]

What will we study next?

1. *Introduction and Preparation*

2. *Analysis with Binary Variables (Chp 1-2)*

3. **Analysis with Multicategory Variables (Chp 3)**

- *3.1 Revisit to Analysis with Contingency Table*
  - *3.1.1 Review of two-way contingency tables*
  - *3.1.2 Analysis of $I \times J$ contingency table*
  - *3.1.3 Multi-way contingency tables (supplementary)*
- **3.2 Analysis with Multicategory Response**