Model Calibration Worksheet

Consider the simple model calibration problem from the notes. We have $n = 5$ field observations in `fieldData.txt` ($[x, y]$) and $m = 20$ computer model runs in `simData.txt` ($[x^*, \theta^*, \eta]$). Here the simulator requires a univariate x^* and a univariate θ^* to produce a univariate output $\eta(x^*, \theta^*)$.

Recall the posterior distribution for θ and other model parameters given in the notes.

Likelihood

$$L(y, \eta | \lambda_\epsilon, \rho_\eta, \lambda_\eta, \lambda_s, \theta) \propto$$
$$|C_{y\eta}|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} \begin{pmatrix} y \\ \eta \end{pmatrix}^T C_{y\eta}^{-1} \begin{pmatrix} y \\ \eta \end{pmatrix} \right\}$$

Priors

$$\begin{aligned}
\pi(\lambda_\epsilon) &\propto \lambda_\epsilon^{a_\epsilon - 1} e^{-b_\epsilon \lambda_\epsilon} \quad \text{perhaps well known from observation process} \\
\pi(\rho_{\eta k}) &\propto \prod_{k=1}^{p_x + p_\theta} (1 - \rho_{\eta k})^{-.5}, \text{ where } \rho_{\eta k} = e^{-.5^2 \beta_k^\eta} \text{ correlation at dist} = .5 \sim \beta(1, .5). \\
\pi(\lambda_\eta) &\propto \lambda_\eta^{a_\eta - 1} e^{-b_\eta \lambda_\eta} \\
\pi(\lambda_s) &\propto \lambda_s^{a_s - 1} e^{-b_s \lambda_s} \\
\pi(\theta) &\propto I[\theta \in C]
\end{aligned}$$

The covariance matrix $C_{y\eta}$ is given by

$$\begin{aligned}
C_{y\eta} &= C^\eta + \begin{pmatrix} \lambda_\epsilon^{-1} I_n & 0 \\ 0 & \lambda_s^{-1} I_m \end{pmatrix} \\
C^\eta &= \lambda_\eta^{-1} R^\eta\left((x, x^*), (\mathbf{1}_m \theta, \theta^*); \rho\right) \\
R^\eta((x, \theta), (x', \theta'); \rho) &= \rho_1^{4(x - x')^2} \times \rho_2^{4(\theta - \theta')^2}
\end{aligned}$$

For this problem, let's pretend we know the values of all of the parameters in the posterior except θ.

ρ	0.21 0.95
λ_η	0.46
λ_ϵ	16
λ_s	80000
C	[0,1]

Code up an MCMC algorithm that samples θ from this posterior density, holding all of the other parameters fixed at the values given above. Note that a Metropolis algorithm will be needed since θ appears in the correlation matrix R^η. What is the posterior mean and 95% interval for θ?