

Collaborations with Randy

Using Folded-Over Designs to Consider Interactions

Arden Miller
Department of Statistics
University of Auckland

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Folded-Over 12-Run Plackett-Burman Design

Original Idea: investigate using the folded-Over 12-Run Plackett-Burman design to consider interactions as well as main effects.

Background

Plackett-Burman designs are 2-levels designs that are typically considered “main effects only” designs.

- main effects are orthogonal to each other.
- complex aliasing between 2-factor interactions and main effects and amongst 2-factor interactions.
- “classical” advice was they should be used when the practitioner was confident factors did not interact.

Analysing Designs with Complex Aliasing

The Hamada and Wu (1992) paper demonstrated that it was possible to investigate interactions using designs (such as the Plackett-Burman designs) that involved complex aliasing.

- Sequential analysis strategy based on empirical principles (effect hierarchy, effect heredity, effect sparsity).
- Worked very well provided that certain conditions were met: only 1 or 2 active 2-factor interactions and main effects dominate.

Analysing Designs with Complex Aliasing

Hamada-Wu procedure:

- 1 Do the main effects only analysis.
- 2 Search for active 2-factor interactions based on the results from 1.
- 3 Re-analyse the main effects if 2 identifies any active 2-factor interactions.

Folded-Over Designs

Folding over is a process applied to 2-level design: for each run in the original design add a run that contains the opposite set of levels.

- Breaks the aliasing between main effects and 2-factor interactions.
- Aliasing amongst main effects and aliasing amongst 2-factor interactions is not affected.
- Doubles the size of the design.

Folded-Over Plackett-Burman Designs

The consequences of folding over a Plackett-Burman design are:

- Main effects are now orthogonal to each other and to 2-factor interactions but there is still complex aliasing between 2-factor interactions.
- Half of the information (df's) are dedicated to main effects and rest to the overall mean (1 df) and 2-factor interactions.

The Folded-Over 12-run Plackett-Burman Design

There are some very attractive features about the folded-over 12-run Plackett-Burman design:

- Run size (24) and capacity (up to 12 factors) are suitable for industrial applications.
- The aliasing pattern is particularly well suited to the Hamada-Wu analysis strategy.
 - increases the number of active 2-factor interactions and the magnitude of these is no longer an issue.

Folded-Over Nonorthogonal Designs

The next step was to look at folding over nonorthogonal main effects plans.

- Started with efficient, minimal, resolution III designs – these designs ensure we get “good” estimates of main effects.
- Evaluated the properties of these design with respect to their ability to identify 2-factor interactions.

Minimal Dependent Sets

Our investigations led us to think about the properties of a design that would determine its ability to differentiate between models.

- Srivastava (1975) concept of resolvability links the ability to differentiate between models to the linear dependencies among the candidate effects.
- Led to our approach of looking at the Minimal Dependent Sets of effects.

Minimal Dependent Sets

A minimal dependent set (MDS) is a linearly dependent set such that if any one element is removed the set becomes independent.

- want the MDS's to be as large as possible as this is directly related to the size of candidate models than can be considered.
- lead us to define MDS pattern, MDS resolution and MDS aberration.

Conclusion