

# Ranking and Prediction for Cycling Canada

by

**Richard R. C. Hsia**

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# Approval

**Name:** Richard R. C. Hsia

**Degree:** Master of Science (Statistics)

**Title:** Ranking and Prediction for Cycling Canada

**Examining Committee:** **Chair:** Jinko Graham  
Professor

**Tim Swartz**  
Senior Supervisor  
Professor

**Joan Hu**  
Supervisor  
Professor

**Dave Clarke**  
External Examiner  
Assistant Professor  
Biomedical Physiology and Kinesiology

**Date Defended:** December 14, 2017

# Abstract

In efforts to improve Canadian performance in the men's Elite UCI Mountain Bike World Cup, researchers from the Canadian Sport Institute Ontario (CSIO) presented to us a specific problem. They had a wealth of race data but were unsure how to best extract insights from the dataset. We responded to their request by building an interactive user interface with R Shiny to obtain rider rankings. Estimation was carried out via maximum likelihood using the Bradley-Terry model. We improved on the existing literature, proposed an exponentially weighted version of the model, and determined an optimal weighting parameter through cross-validation involving performance of future races. Therefore, the proposed methods provide forecasting capability. The tuned Bradley-Terry estimation performed better than the UCI point-based ranking in terms of predictive error. This implementation of the Bradley-Terry model with a user-friendly graphical interface provides broader scientific audiences easy access to Bradley-Terry ranking for prediction in racing sports.

**Keywords:** cycling; ranking; prediction; Bradley-Terry; pairwise-comparisons

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# Chapter 1

## Introduction

Researchers from Canadian Sport Institute Ontario (CSIO) approached us to discuss how we might be able to contribute to their statistical analysis efforts. CSIO is a national sports centre dedicated to putting more Canadian athletes on the international podium by applying sport science, sport medicine, athlete/coach and staff development. The institute is heavily invested in Canadian performance at Olympic, Paralympic Games and World Championships. For example, CSIO is a vital partner of the renowned Own the Podium organization.

### 1.1 Motivation

For the last few years, CSIO scientists have been involved with Cycling Canada, a national sport organization for the promotion of cycling in Canada. CSIO has been providing performance analysis support at various international competitions, such as the Mountain Bike World Cup held by Union Cycliste Internationale (UCI). This series is composed of a combination of cross-country and downhill disciplines, each with an Elite Men and Elite Women category. Athletes compete in several rounds during each season, with points awarded according to their placing in each event.

Much of Cycling Canada's recent focus has been on Canadian performance in the Elite cross-country events. While Canadian women routinely win or reach the podium, the men's podium has been dominated by the Swiss rider Nino Schurter and Frenchman Julien Absalon for the past 15 years. They share a combined 12 series winner titles from 2003 to 2017. The last Canadian man to win the cross-country series was Roland Green in 2001.

To assist in their cycling analytics endeavours, CSIO presented to us a specific problem. Their researchers and coaches received race data in a particular format not conducive to manipulation and inference. They were also unsure how to best utilize the available data to obtain insights. We were tasked to design a clean, intuitive user interface to perform rider ranking and predictive analyses “under the hood” for the men's cross-country events.

CSIO researchers did not believe that individual race times were noteworthy, and suggested that they should not form part of the analysis. Thus, the baseline model is a multinomial distribution on  $n!$  permutations of race order, where  $n$  is the number of racers. However, the observations are sparse because races are few compared to the number of permutations of race order. For example, in a set of 10 races with 80 racers each, we only observe a maximum of 10 of the possible  $80!$  orderings. There would be no observations for the other  $80! - 10$  possible orderings, therefore we would have no idea of their probabilities. Furthermore, not every cyclist competes in every race. Therefore, some alternative is needed, such as a reduced model or data reduction.

## 1.2 Literature review

In recent years, many approaches have been implemented to perform ranking and prediction on data across a variety of sports [1, 4, 6, 7, 9]. The Bradley-Terry model [2] is one of the most commonly applied models, especially in match-based team sports such as basketball, where one team competes with only one other team at the same time [4]. In each paired-comparison, there is a clear winner and loser. The Bradley-Terry model treats the outcome of each comparison as an independent Bernoulli random variable.

While paired-comparison methods such as the Bradley-Terry model have been widely applied to produce rankings in team sports, their use has been limited in racing sports. This scarcity may perhaps be because paired-comparisons are not as obvious in racing sports as in team sports. Anderson [1] constructed a Bradley-Terry model to produce a ranking for Formula One racers, the highest class of single-seat auto racing, where he compared  $\binom{n}{2}$  pairs per race. This may have been in error, as it is obvious that the outcomes of such  $\binom{n}{2}$  paired-comparisons are not independent, which is a fundamental assumption of the Bradley-Terry model. This may explain why Anderson's findings expressed some unusual probabilistic results.

Due to the shortage of quality publications on Bradley-Terry implementation in racing sports, especially bicycling, the UCI Mountain Bike World Cup dataset presented a unique challenge. Most racing sports including Formula One and the Mountain Bike World Cup employ a point-based ranking system. That is, competitors accumulate points throughout a season based on their performances in individual races. Competitors and fans alike often misleadingly interpret these rankings as measures of ability.

In many racing sports, points are not awarded to the bottom half of the finishers. For example, in Formula One, points are only awarded to the top ten finishers in decreasing increments. The drivers out of the top ten do not receive any points, and would thus be ranked equally, which would not be representative of their true abilities. Further, not all competitors compete in all the races throughout a season. A point-based ranking system would favour mediocre but reliable performance over a few spectacular race outcomes. These

are just a few of the many issues that may exist with interpreting point-based rankings as measures of ability.

### 1.3 Approach

The Bradley-Terry model remedies these issues by ranking competitors based on their estimated true abilities. In this project, we aimed to provide a reliable implementation for producing rankings in racing sports accessible to broader scientific audiences, with the intention of using the rankings for prediction in future races. We achieved this by creating a Shiny application that analyses a variety of inputs, allows user specification of certain parameters, and generates a Bradley-Terry ranking. The algorithms implemented include a basic Bradley-Terry model and an exponentially weighted version which allows for a user-defined weighting constant.

Consider a race with cyclists  $i_1, i_2, \dots, i_n$  where we are interested in the probability  $P(i_1 > i_2 > \dots > i_n)$ . The probabilistic notation specifies the finishing order of the cyclists where  $i_j > i_{j+1}$  denotes that cyclist  $j$  defeated cyclist  $j + 1$ . In our Bradley-Terry implementation, we have  $n - 1$  comparisons that correspond to the probabilities  $P(i_1 > i_2), P(i_2 > i_3), \dots, P(i_{n-1} > i_n)$ . Therefore, there is an implicit assumption that  $P(i_1 > i_2 > \dots > i_n) = P(i_1 > i_2) \times \dots \times P(i_{n-1} > i_n)$ . Similar to the assumption made by Anderson [1], our assumption is incorrect, but “not as incorrect” as Anderson [1]. The difficulty in our assumption can be seen by expressing

$$P(i_1 > i_2 > \dots > i_n) = P(i_1 > i_2 \mid i_2 > i_3 > \dots > i_n)P(i_2 > i_3 > \dots > i_n).$$

It seems to us that  $P(i_1 > i_2 \mid i_2 > i_3 > \dots > i_n) \neq P(i_1 > i_2)$ , which is the underlying premise of the Bradley-Terry model as paired outcomes are assumed independent.

We believe that  $P(i_1 > i_2 \mid i_2 > i_3 > \dots > i_n) \neq P(i_1 > i_2)$  since the condition  $i_2 > i_3 > \dots > i_n$  indicates that  $i_2$  has performed very well in the race. We therefore expect  $P(i_1 > i_2 \mid i_2 > i_3 > \dots > i_n) < P(i_1 > i_2)$ . The expected consequence of this assumption is that the resulting probabilities obtained by the Bradley-Terry estimation algorithm may be more extreme for the top-end racers in the positive sense and more extreme for the bottom-end racers in the negative sense. We do not plan on using existing Bradley-Terry inferential capabilities (e.g. the construction of confidence intervals). Rather, we view the Bradley-Terry algorithm applied to the  $n - 1$  comparisons as a black-box procedure that produces ranks. Later, we introduce a cross-validation procedure that tests the rankings in terms of future race prediction. This is done by training on a subset of the data to determine optimal weighting constants based on mean absolute error (MAE) and root mean squared error (RMSE).

Chapter 2 describes and provides further context for the Mountain Bike World Cup data. Chapter 3 outlines the statistical models. Chapter 4 presents predictions and results. Chapter 5 offers a brief discussion on findings and a discussion of future research.

## Chapter 2

# Data

CSIO researchers presented us with a dataset containing four years of UCI Mountain Bike World Cup races. The data spanned from 2011-2014 and covered 32 races. The data included race results for all the races held in the Elite, Junior, and U23 leagues in the cross-country discipline, totalling 7,625 rows. Other descriptors included race level, race date, race location, rider name, rider nationality, and UCI unique identifier code. As previously mentioned, CSIO did not believe that race times were important and excluded such information from the dataset.

After the end of every season, a special World Championship race is held. This race differs from World Cup season races in many aspects. For example, the competitors in the World Championships represent national teams rather than commercial teams. This leads to coaching, training, and strategical differences compared to World Cup races. In the Championship race, a wider range of riders are invited. This leads to the phenomenon where at least 10% of the riders in the Championship race have only participated in past championship races, and have never raced in any of the season races. Furthermore, the results of the Championship race do not count toward a racer's season point-based ranking. CSIO researchers were interested in World Cup ranking and prediction. Therefore, of the 32 races included in the dataset, only the 28 World Cup races were of analytical focus.

The data arrived from CSIO as an Excel spreadsheet (.xlsx) in a highly vertical format where each row only contained one race result for one racer (Table 2.1). Our algorithm converted the data from this redundant format into one that was more concise and conducive to analysis where each row contained all the race results for one racer (Table 2.2). If a racer did not compete in or finish a race, the entry was coded with a 0. The graphical user interface for the algorithm also provided users the ability to choose the specific league and races to be included in the data conversion. Four of the original columns were unnecessary for our analyses and were not included in the data format conversion.

The Mountain Bike World Cup season typically runs from April through August, with the number of races each season ranging from six to eight. The aforementioned World Championship race is held after each season, typically in September. The Mountain Bike

race_level	RACE_DATE	RACE_LOCATION	RIDER_CLASS	GENDER	RIDER	FINAL_RANK	COUNTRY	UCI_CODE
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Absalon Julien	1	France	FRA19800816
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Schurter Nino	2	Switzerland	SUI19860513
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Fontana Marco Aurelio	3	Italy	ITA19841012
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Milatz Moritz	4	Germany	GER19820624
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Fumic Manuel	5	Germany	GER19820330
World Championships	6-Sep-14	Hafell, Norway	Elite	Men	Mantecon Gutierrez Sergio	6	Spain	ESP19840925

Table 2.1: Example of Excel data from CSIO.

Riders	13-Apr-14	27-Apr-14	24-May-14	31-May-14	3-Aug-14	10-Aug-14	24-Aug-14	6-Sep-14
Absalon Julien	1	1	5	1	2	2	2	1
Fumic Manuel	2	20	11	7	11	6	3	5
Marotte Maxime	3	3	15	9	42	18	15	17
Cink Ondrej	4	8	46	0	0	17	22	12
Hermida Ramos José Antonio	5	5	6	14	8	16	10	9
Schurter Nino	6	0	1	2	1	1	1	2

Table 2.2: Example of data format after conversion.

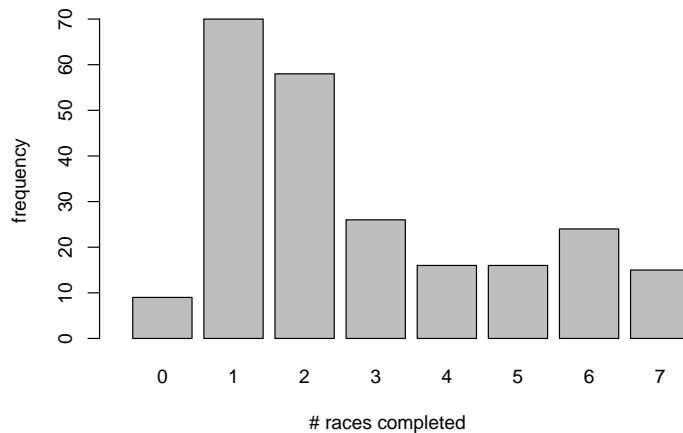


Figure 2.1: A histogram of the number of Elite competitor race finishes for the 2014 Mountain Bike World Cup season.

World Cup series is an international event with races being held in cities around the world. For example, the 2014 season featured races held in South Africa, Australia, Czech Republic, Germany, Canada, United States, France, and Norway. The wide variety of race factors including event elevation, climate, and surroundings that racers experience can have an impact on race performance.

Furthermore, not all riders compete and finish in every event of the season. Many do not participate in races due to injury, some are substitute riders racing in a single race, and others may not finish a race due to mechanical failures and punctures. Therefore, not all racers face each other for an equal number of races. For example, of the 233 Elite racers who competed in the 2014 season, 70 riders finished only one out of the seven races. Meanwhile, only 16 racers completed all seven races of the season, including the season champion Julien Absalon. The result is a heavily right-skewed distribution of race finishes (Figure 2.1). For comparison, 13 Canadian men competed in the 2014 season. The competitor with the most finishes was Raphael Gagne with six, while the majority of Canadian riders finished only two races (Figure 2.2).

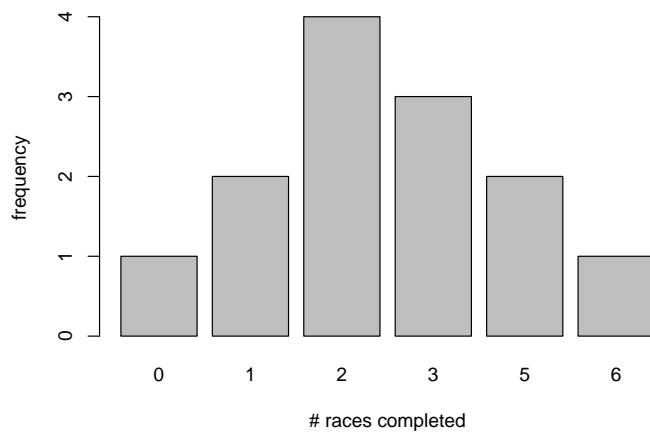


Figure 2.2: A histogram of the number of Canadian Elite competitor race finishes for the 2014 Mountain Bike World Cup season.



# Chapter 3

## Models

The estimation of competitors' true abilities was carried out via maximum likelihood estimation using the Bradley-Terry model.

### 3.1 Formulation

Suppose that competitor  $j$  possesses a latent ability score  $\alpha_j$  representing their inherent “strength”, which is assumed to be constant across the analysed races. The performance of rider  $j$  in race  $r$  is subject to a variety of stochastic factors  $\varepsilon_{rj}$  including but not limited to those mentioned in Chapter 2. The Bradley-Terry model assumes that the  $\varepsilon_{rj}$ 's are identically and independently distributed standard Gumbel random variables with cumulative distribution function

$$F(\varepsilon_{rj}) = \exp[-\exp(-\varepsilon_{rj})] \quad -\infty < \varepsilon_{rj} < \infty. \quad (3.1)$$

The standard Gumbel is a special case of the two parameter Gumbel distribution (also known as the log-Weibull distribution and the double exponential distribution). The Gumbel distribution has cumulative distribution function

$$F(x) = \exp[-\exp(-(x - \mu)/\beta)] \quad -\infty < x < \infty,$$

where  $-\infty < \mu < \infty$  and  $\beta > 0$ . Properties of the Gumbel distribution include the mode  $\mu$ , the mean  $\mu + \gamma\beta$  where  $\gamma \approx 0.5772$  is the Euler-Mascheroni constant, and standard deviation  $\beta\pi/\sqrt{6}$ . The standard Gumbel distribution corresponds to the case where  $\mu = 0$  and  $\beta = 1$ . Figure 3.1 shows the density functions of various Gumbel distributions.

The resulting performance for rider  $j$  in race  $r$  can then be expressed as

$$m_{rj} = \alpha_j + \varepsilon_{rj}. \quad (3.2)$$

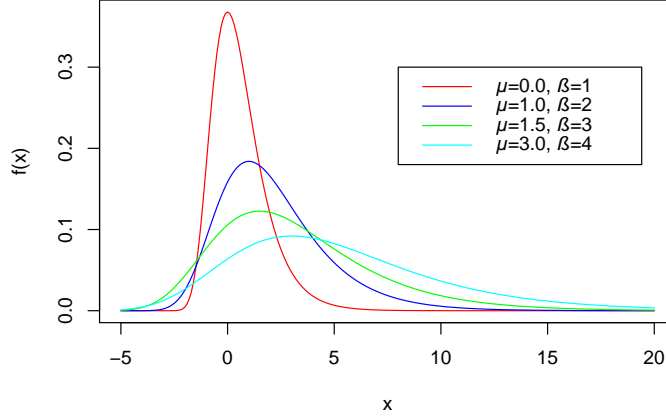


Figure 3.1: Probability density function of the Gumbel distribution with different parameters.

The continuous performance variable  $m_{rj}$  is latent (i.e. unobserved) and in the context of cycling could be interpreted as the time to complete the race or some transformation of time. Combining Equations (3.1) and (3.2), the distribution of  $m_{rj}$  is also a Gumbel random variable with distribution function

$$F(m_{rj}) = \exp[-\exp(-(m_{rj} - \alpha_j))].$$

The difference in performance between riders  $j$  and  $k$  is given by  $m_{rj} - m_{rk}$  which is simply a difference between two Gumbel random variables. The difference follows a logistic distribution with distribution function

$$F(m_{rj} - m_{rk}) = \frac{1}{1 + \exp[-((m_{rj} - m_{rk}) - (\alpha_j - \alpha_k))]}.$$

Thus, the probability that rider  $j$  beats  $k$  is

$$\begin{aligned} P(m_{rj} - m_{rk} > 0) &= 1 - \frac{1}{1 + \exp[-((m_{rj} - m_{rk}) - (\alpha_j - \alpha_k))]} \\ &= \frac{\exp(\alpha_j - \alpha_k)}{1 + \exp(\alpha_j - \alpha_k)} \\ &= \frac{\exp(\alpha_j)}{\exp(\alpha_j) + \exp(\alpha_k)}. \end{aligned}$$

The latent ability score could then be interpreted as a measure of the competitor's relative tendency of beating another competitor, averaged over a set of races. It can be observed here that the  $\alpha_j$ 's are not identifiable. That is,

$$\alpha'_j = \alpha_j + c \quad \forall j$$

gives the same likelihood, where  $c$  is a constant. Therefore, the likelihood function for the Bradley-Terry model is

$$L(\alpha) = \prod_{(j,k)} \frac{\exp(\alpha_j)}{\exp(\alpha_j) + \exp(\alpha_k)} \quad (3.3)$$

where the product is taken over a set of paired-comparisons in race  $r$ . As previously discussed in Section 1.3, the set was chosen as the  $n - 1$  sequential race order comparisons.

## 3.2 Ability estimation

Estimation of the  $\alpha$ 's is then a maximum likelihood problem, where the values of  $\alpha$  are sought that maximize the probability of the observed race outcomes

$$(i_1 > i_2), (i_2 > i_3), \dots, (i_{n-1} > i_n).$$

Recall again that the sequential ordering of pairwise-comparisons implies an independence that may not be true. Many iterative optimization techniques may be employed to estimate these parameters [4, 6].

Here, we consider a simple approach. Let the probability that rider  $j$  beats  $k$  in race  $r$  be simplified as

$$p_{rjk} = P(m_{rj} - m_{rk} > 0) = \frac{\exp(\alpha_j)}{\exp(\alpha_j) + \exp(\alpha_k)}.$$

The Bradley-Terry likelihood (3.3) can then be expressed as

$$\begin{aligned} L(\alpha) &= \prod_{(j,k)} p_{rjk} \\ &= \prod_{i=1}^{n-1} (p_{rjk})^{y_i} (1 - p_{rjk})^{1-y_i}, \end{aligned} \quad (3.4)$$

where  $\mathbf{y} = (y_1, \dots, y_{n-1})^T$  is a vector of 1's corresponding to the sequential paired comparisons. Notice that (3.4) is now identical to the likelihood function for logistic regression. This renders a convenient way to implement the algorithm in R with `glm` [1, 8, 9].

Specifically, estimation is achieved with `glm(Y~X-1, family=binomial(logit))`. Note that `X-1` denotes regression without the intercept term. Here,  $\mathbf{Y}$  is a vector of ones  $\mathbf{y}$ , and  $\mathbf{X}$  is a data matrix  $\mathbf{X}$ . This  $\mathbf{X}$  matrix is defined such that each column corresponds to a racer  $1, \dots, n$  and each row corresponds to a paired-comparison. Each row records the outcome of one paired-comparison, assigning 1 to the winner and  $-1$  to the loser, with zeroes everywhere else. The resulting matrix contains only two non-zero values in each row. Riders who failed to finish a race were assigned zeroes for all of their column values in that race. Finally, one competitor should be removed from the regression to act as a baseline. This is done by setting that racer's column values all to zeroes. The estimated ability  $\alpha$  of

this competitor would be zero, with all the other racers' abilities relative to the baseline rider. The difference of the estimated  $\alpha$ 's between riders are unaffected by the rider chosen as baseline.

To illustrate, consider a set of three races and  $n = 5$  competitors A, B, C, D, and E in each race. Let the finishing order in the first race be A, B, C, D, E, the finishing order in the second race E, D, C, B, A, and in the third race A, C, E, B, with rider D failing to finish. Thus, the corresponding  $\mathbf{X}$  would resemble

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{r=1} \\ \mathbf{X}_{r=2} \\ \mathbf{X}_{r=3} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \text{race} \\ & 1 \\ \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix} & \text{race} \\ & 2 \\ \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix} & \text{race} \\ & 3 \end{bmatrix} .$$

Note that baseline rider E is removed from the analysis by setting its column values to zero. Also note that  $\mathbf{X}_{r=3}$  has three rows because only four riders finished the race.

The resulting estimated coefficients correspond to estimates of the  $\alpha$ 's and can be extracted with `model$coefficients`.

### 3.3 Weighted Bradley-Terry

In order to explore the time series effects on rider ability estimates, we developed an exponentially weighted version of the classical Bradley-Terry model. We believe that we are the first to do this in the context of Bradley-Terry models in racing sports. Let

$$w_t = \exp(-\theta t) \tag{3.5}$$

be the weighting factor, where  $\theta \geq 0$  is a tuning parameter and

$$t = r_{\max} - r$$

is the number of races prior to the most recent one. Thus, the most recent race would correspond to  $t = 0$  and have a weighting factor of  $w_t = 1$ . The weights  $w_t$  for  $t > 0$  are

decreasing in  $t$ . Note that although we have used the variable  $t$  to suggest time, the races are not necessarily evenly spaced and therefore  $t$  is not a true time variable.

This is analogous to the well-known technique of exponential smoothing for time series data, first introduced by Brown [3]. Exponential smoothing is often presented in time series applications as a geometric progression  $1, (1 - a), (1 - a)^2, \dots, (1 - a)^{t_{\max}}$ , where  $0 < a < 1$  is the smoothing factor. This geometric series is practically a discrete version of the exponential decay function (3.5). The rationale behind exponential weights is the belief that older observations may be less relevant for prediction than more current ones.

Instead of assigning 1 and  $-1$  respectively for the winner and loser of each paired-comparison at time  $t$ , the winner is assigned  $w_t$  and the loser  $-w_t$ . This regime assigns decrementing weights to races further back in time. If  $\theta = 0$ ,  $w_t = 1$  and the model returns to that of the classical Bradley-Terry model, weighting all races equally. Suppose  $\theta = 0.1$ . Then in a season with eight races, the weights would be  $w_0, \dots, w_7 = 1.00, 0.90, 0.82, 0.74, 0.67, 0.61, 0.55, 0.50$ . That is, a win accumulated in the first race of the season would be worth half that of a win in the eighth race. Likewise, a loss would only have half the impact on the Bradley-Terry ranking compared to a loss in the eighth race. Now suppose  $\theta = 1$ , then  $w_0, \dots, w_7 = 1.000, 0.368, 0.135, 0.050, 0.018, 0.007, 0.002, 0.001$  which quickly leads to vanishing weights. Naturally, the next step is to determine an optimal value of  $\theta$ .

To illustrate, consider again a set of three races and  $n = 5$  competitors A, B, C, D, and E in each race. Let the finishing order in the first race be A, B, C, D, E, the finishing order in the second race E, D, C, B, A, and in the third race A, C, E, B, with rider D failing to finish. For weighting, let  $\theta = 0.1$ . Thus, the corresponding  $\mathbf{X}$  would resemble

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{r=1} \\ \mathbf{X}_{r=2} \\ \mathbf{X}_{r=3} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{t=2} \\ \mathbf{X}_{t=1} \\ \mathbf{X}_{t=0} \end{bmatrix} = \begin{array}{ccccc} & \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \\ \left[ \begin{array}{ccccc} 0.82 & -0.82 & 0 & 0 & 0 \\ 0 & 0.82 & -0.82 & 0 & 0 \\ 0 & 0 & 0.82 & -0.82 & 0 \\ 0 & 0 & 0 & 0.82 & 0 \end{array} \right] & & & & & \text{race} \\ & & & & & & 1 \\ \left[ \begin{array}{ccccc} 0 & 0 & 0 & -0.90 & 0 \\ 0 & 0 & -0.90 & 0.90 & 0 \\ 0 & -0.90 & 0.90 & 0 & 0 \\ -0.90 & 0.90 & 0 & 0 & 0 \end{array} \right] & & & & & \text{race} \\ & & & & & & 2 \\ \left[ \begin{array}{ccccc} 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{array} \right] & & & & & \text{race} \\ & & & & & & 3 \end{array}$$

Again, note that baseline rider E is removed from the analysis by setting its column values to zero. Also note that  $\mathbf{X}_{r=3}$  has three rows because only four riders finished the race.

While the Bradley-Terry model assumes that competitor strengths are constant, our weighted approach addresses this constraint and recognizes that strengths may vary through time. We are also aware of other methods allowing for time-varying abilities such as the dynamic Bradley-Terry or Bayesian models where  $\alpha_j$ 's update after every race.

### 3.4 Cross-validation

An optimal value of  $\theta$  can be determined through cross-validation. The two-step process involved estimating an optimal  $\hat{\theta}$ , then using  $\hat{\theta}$  to validate performance.

Over the four years and 28 World Cup races worth of results, we determined an optimal  $\hat{\theta}$  based on using races  $1, \dots, 26$  to predict race 27. Our analysis considered only racers who participated in the 27th race and also at least one of the first 26, i.e.

$$\{\text{analysed riders}\} = \{\text{riders in first 26 races}\} \cap \{\text{riders in 27th race}\}.$$

From the subset of intersecting riders, an  $\mathbf{X}$  matrix was constructed based on the sequentially ordered paired-comparison outcomes from the first 26 races. Considering testing  $\theta$  at 0.01 intervals from 0 to 1,

$$\theta_1, \dots, \theta_{101} = 0.00, 0.01, \dots, 0.99, 1.00.$$

We created 101  $\mathbf{X}$  matrices each with the corresponding weighting factors  $w_t$  as described in (3.5). Rankings were obtained by performing Bradley-Terry estimation on each of the  $\mathbf{X}$ 's. Then, riders were ranked according to their estimated strength parameters  $\hat{\alpha}_j$ .

For each  $\theta$ , every rider was assigned a predicted ranking based on the Bradley-Terry ability estimation results. For each  $\theta$ , rider  $j$ 's predicted ranking  $\hat{i}_j$  for the 27th race was compared with their actual ranking  $i_j$  in the 27th race. Model accuracy was assessed with the mean absolute error

$$\text{MAE} = \frac{1}{n} \sum_{j=1}^n |i_j - \hat{i}_j|$$

and the root mean squared error

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{j=1}^n (i_j - \hat{i}_j)^2}.$$

An optimal  $\hat{\theta}$  was chosen that minimized both MAE and RMSE.

With the optimal  $\hat{\theta}$ , we validated the tuned model by using races  $1, \dots, 27$  to predict race 28. This time, our analysis considered only racers who participated in the 28th race and racers for whom rankings were predicted. Our procedure to exclude race 28 from estimation

of  $\hat{\theta}$  was based on the principle that data should not be used both for estimation and validation. Finally, model performance was measured with MAE and RMSE for validation.

## Chapter 4

# Predictions and results

Our approach provides a reliable implementation of Bradley-Terry ranking for Cycling Canada based on two stages. First, we determined an optimal value of  $\theta$  through cross-validation, as described in Section 3.4. Then, we designed a Shiny application which provided a graphical user interface for producing predictions using cross-validation.

### 4.1 Cross-validation and predictions

Using races  $1, \dots, 26$  to predict race 27, the data matrix  $\mathbf{X}$  had 997 rows and 89 columns. That is, our analysis considered the 89 racers who participated in the 27th race and also at least one of the first 26. Then, 997 sequential paired-comparisons were constructed based on the race results of the 89 racers. Testing  $\theta$  at 0.01 intervals from 0 to 1, we constructed 101  $\mathbf{X}$  matrices with different  $\theta$ 's.

Performing Bradley-Terry estimation on each of the  $\mathbf{X}$ 's produced “weighted” rankings for each  $\theta$ . Comparing the weighted rankings with the actual ranking in the 27th race, the MAE and RMSE were calculated for each  $\theta$ . Figure 4.1 shows the relationship between  $\theta$  and MAE and RMSE. There is some variation in the plots, but they are not representative of real trends. Therefore, a lowess curve with a bandwidth of  $1/3$  is applied as a smoother for visualization.

*A priori*, we did not know the range of  $\theta$  leading to good predictions (i.e. small MAE and RMSE). It is apparent from Figure 4.1 that the interesting portion of the plots lies in the interval  $[0.0, 0.1]$ . We therefore repeated the cross-validation exercise testing 101  $\theta$  values equally spaced in the range  $[0.0, 0.1]$ . The resulting plots are given in Figure 4.2.

Based on the lowess curve,  $\theta = 0.075$  produced the lowest MAE of 10.1 while  $\theta = 0.073$  produced the lowest RMSE of 14.5 in predicting race 27. Therefore, the midpoint  $\hat{\theta} = 0.074$  was determined to be the optimal value. This corresponds to weights  $w_0, \dots, w_{25} = 1.00, \dots, 0.16$ , as illustrated in Figure 4.3. The actual race results and predicted ranking at  $\hat{\theta} = 0.074$  for the top 20 finishers in race 27 are shown in Table 4.1.



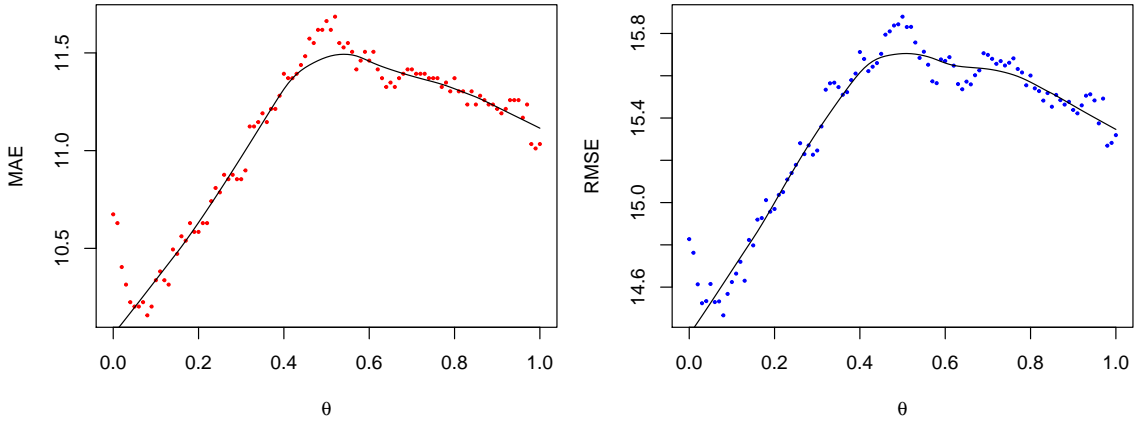


Figure 4.1: Scatterplot of weighting parameter  $0 < \theta < 1$  and MAE and RMSE. The black line is a corresponding loss curve.

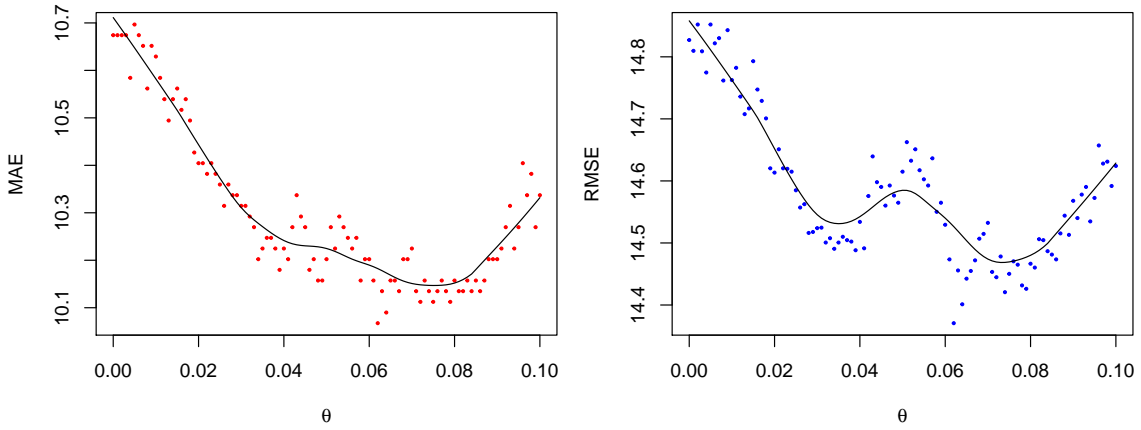


Figure 4.2: Scatterplot of weighting parameter  $0.0 < \theta < 0.1$  and MAE and RMSE. The black line is a corresponding loss curve.

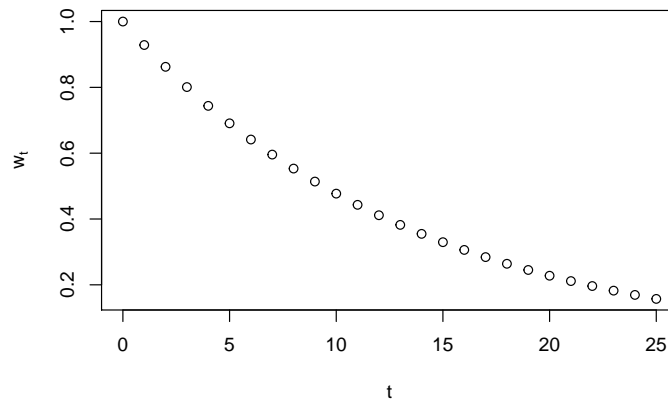


Figure 4.3: Scatterplot of weighting factor  $w_t$  and the number of races  $t$  prior to the most recent race when  $\hat{\theta} = 0.074$ .

Rider	Actual	Ranking $\hat{\theta} = 0.074$	Difference
Schurter Nino	1	1	0
Absalon Julien	2	2	0
Flückiger Lukas	3	12	-9
McConnell Daniel	4	6	-2
Flückiger Mathias	5	9	-4
Fumic Manuel	6	8	-2
Mantecon Gutierrez Sergio	7	13	-6
Tempier Stéphane	8	5	3
Vogel Florian	9	11	-2
Näf Ralph	10	17	-7
Giger Fabian	11	14	-3
Kabush Geoff	12	18	-6
Zandstra Derek	13	52	-39
Van Houts Rudi	14	20	-6
Fontana Marco Aurelio	15	16	-1
Hermida Ramos José Antonio	16	4	12
Cink Ondrej	17	7	10
Marotte Maxime	18	15	3
Lindgren Emil	19	19	0
Tiberi Andrea	20	23	-3

Table 4.1: Actual results and ranking ( $\hat{\theta} = 0.074$ ) for the top 20 finishers of race 27.

With the optimal  $\hat{\theta} = 0.074$ , we continued the analysis by using races 1,  $\dots$ , 27 to predict race 28. The data matrix  $\mathbf{X}$  had 1051 rows and 88 columns. That is, our analysis considered the 88 racers who participated in the 28th race and also in at least one of the first 27. Then, 1051 sequential paired-comparisons were constructed based on the race results of the 88 racers.

For validation, Bradley-Terry estimation was performed on the  $\mathbf{X}$  with  $\hat{\theta} = 0.074$ . The predicted rankings (Table 4.2) produced MAE and RMSE of 12.4 and 18.0, respectively. These two values are higher than what was obtained in Figure 4.2, which emphasizes the importance of separating estimation from validation. The MAE of 12.4 corresponds to predictive ranking errors on average of around 12 positions. However, the MAE of the top 20 racers is only 6.6, which is considerably smaller than that of all 88 riders. The MAE further reduces to 4.3 when considering the top 10 racers. This is likely due to the stratification in true rider abilities. That is, the top riders' performances are much more consistent and superior than the rest of the field.

For example, it can be observed that Luca Braidot finished race 28 with a result (16) much higher than predicted (59). This may seem surprising at first, but taking a look at his race history provides some explanation. Braidot raced in the U23 league prior to the 2014 season, and performed well in the 2013 season with three top five finishes. Recently having been upgraded to the Elite league, his best finish was in 17th, but his consistency

Rider	Actual	Ranking	Difference
		$\hat{\theta} = 0.074$	
Schurter Nino	1	1	0
Absalon Julien	2	2	0
Fumic Manuel	3	8	-5
Flückiger Lukas	4	5	-1
Kerschbaumer Gerhard	5	20	-15
McConnell Daniel	6	6	0
Näf Ralph	7	17	-10
Flückiger Mathias	8	4	4
Vogel Florian	9	10	-1
Hermida Ramos José Antonio	10	3	7
Lindgren Emil	11	21	-10
Milatz Moritz	12	15	-3
Giger Fabian	13	14	-1
Mantecon Gutierrez Sergio	14	11	3
Marotte Maxime	15	12	3
Braidot Luca	16	59	-43
Fanger Martin	17	24	-7
Fontana Marco Aurelio	18	13	5
Van Houts Rudi	19	18	1
Ettinger Stephen	20	32	-12

Table 4.2: Actual results and ranking ( $\hat{\theta} = 0.074$ ) for the top 20 finishers of race 28.

suffered. In race 26 and 27 he finished in 105th and 50th, respectively. It is apparent that Braidot had the ability to finish in the top 20, but his recent races were disappointing. Due to the weighting of the Bradley-Terry estimation, his earlier good performances were downweighted compared to the recent poor outcomes.

## 4.2 Point-based ranking comparison

Further insight on field stratification may be gained by studying the UCI point-based ranking. As mentioned in Section 1.1, the series has long been dominated by the riders Nino Schurter and Julien Absalon. Their performances have been so commanding that according to the point-based system (Table 4.3), other racers in the top ten fail to even accumulate half of their points in a season (Table 4.4). For example, series winner Julien Absalon and runner-up Nino Schurter respectively accrued 1490 and 1330 points in the 2014 season. However, fifth place Stéphane Tempier came away with only 785 points. Rider points quickly dropped off from there.

However, as discussed in Section 1.2, many issues exist with interpreting the rankings as measures of athlete ability in point-based rankings of racing sports. Thus, predicting future performance based on point-based rankings may produce misleading results. The Bradley-Terry model remedies these concerns by ranking competitors based on their estimated true

Pos	Pts	Pos	Pts	Pos	Pts	Pos	Pts	Pos	Pts	Pos	Pts
1	250	11	90	21	64	31	44	41	27	51	17
2	200	12	85	22	62	32	42	42	26	52	16
3	160	13	80	23	60	33	40	43	25	53	15
4	150	14	78	24	58	34	38	44	24	54	14
5	140	15	76	25	56	35	36	45	23	55	13
6	130	16	74	26	54	36	34	46	22	56	12
7	120	17	72	27	52	37	32	47	21	57	11
8	110	18	70	28	50	38	30	48	20	58	10
9	100	19	68	29	48	39	29	49	19	59	9
10	95	20	66	30	46	40	28	50	18	60	8

Table 4.3: UCI Mountain Bike World Cup cross-country points system.

Pos	Athlete	Points
1	Julien Absalon	1490
2	Nino Schurter	1330
3	Daniel McConnell	970
4	Manuel Fumic	856
5	Stéphane Tempier	785
6	Mathias Fluckiger	785
7	José Antonio Hermida	767
8	Lukas Fluckiger	709
9	Sergio Mantecon Gutierrez	683
10	Maxime Marotte	668

Table 4.4: Top ten riders at the end of the 2014 season according to the UCI point-based ranking.

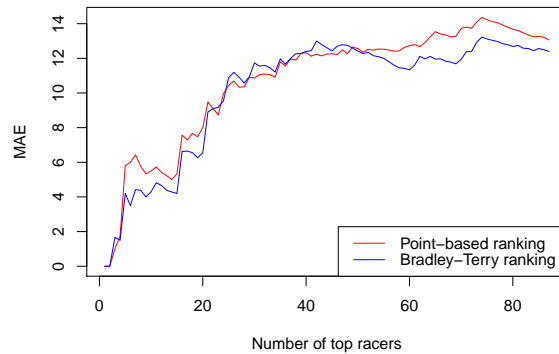


Figure 4.4: Relationship between the number of top racers considered and the MAEs of the Bradley-Terry and point-based rankings.

abilities. To demonstrate, we took the same 88 racers from the validation step and ranked them according to their UCI point-based ranking prior to race 28.

Prediction of the 28th race based on these rankings produced MAE of 13.1. The MAE was 8.0 for the top 20 racers and 5.5 for the top 10. Comparing with the results in Section 4.1, Bradley-Terry prediction of the same race produced MAE of 12.4. The Bradley-Terry prediction MAE was 6.6 for the top 20 racers and 4.3 for the top 10. Evidently, the Bradley-Terry model performed better in terms of lower MAE than the point-based rankings in predicting race 28 results, especially for the top racers. Figure 4.4 shows the relationship between the number of top racers considered and the MAEs of the Bradley-Terry and point-based rankings.

### 4.3 App for Cycling Canada

Shiny is an R package for building interactive apps for data analysis. It allows statistical computing to be performed behind the scenes in R, while interacting with broader scientific audiences through a graphical user interface. Therefore, the user need not understand the code nor model intricacies in order to use the app and benefit from the results.

The code for our app was written in a single .R script file with both the user interface and server components. The user interface allows users to specify a host of input parameters. To begin, the user would choose a CSV file from their computer (Figure 4.5). In the case of the Cycling Canada Excel spreadsheets, they had to be converted to CSV prior to selection, a very simple procedure.

As presented in Chapter 2, the dataset contained four years of race results for all the races held in the Elite, Junior, and U23 leagues in the cross-country discipline, totalling 7,625 rows. This includes both World Cup and World Championship races. If the file is successfully uploaded in the correct format, the selection screen expands to show further options as shown in Figure 4.6. The user could filter the dataset by league, race type, and

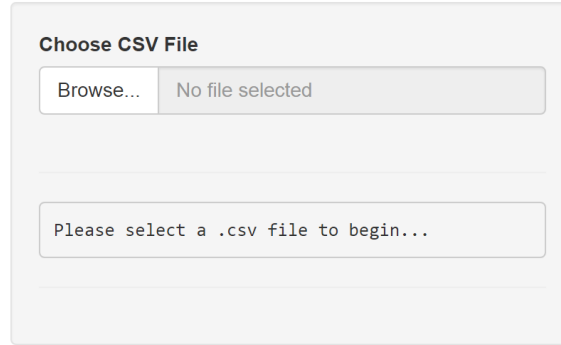


Figure 4.5: Shiny user interface for selecting CSV input file.

race date. The application automatically scans the dataset for league, race type, and race date values and presents them in drop-down menus.

If valid filters are applied, the interface (below the selection inputs) would show the number of riders and races in the selected data subset (Figure 4.7). These values could be used for user information or data verification. A preview of the subset is also presented to the right of the options bar. The data preview allows the user to further sort and search the subset for data verification.

For example, Figure 4.7 shows the user interface after selecting the 2014 World Cup season of the Elite league. It can be seen that the data summary shows 233 riders and seven races, matching that of the example in Chapter 2. This assures the user that the data was read correctly. Furthermore, the right side of the user interface shows a preview of the selected data for this subset.

Upon verifying the selected data, the user has the option of specifying a weighting parameter  $\theta$  in the numeric entry field at the bottom of the options bar. The default value is set to 0, corresponding to the classical Bradley-Terry model, weighting all races equally. Of course, our cross-validation procedure recommends  $\theta = 0.074$ . Once satisfied with the chosen options, the user could press on the “Run Bradley-Terry Ranking” button and the application would perform the requested analysis in the background.

Depending on the size of the dataset to be analysed, analysis typically takes several seconds to a few minutes to complete. Throughout the process, a progress bar at the bottom right of the window provides details on the task being carried out (Figure 4.8). If for any reason the analysis needed to be stopped prior to completion, the user could simply press on the “X” in the progress bar to abort the process. Upon successful completion of the Bradley-Terry estimation, a table of predicted rankings is presented to the right of the options bar, replacing the data preview table (Figure 4.9). Similarly, this table is sortable and searchable. Note that some rider names have accented letters rendered incorrectly. This is an artefact originating from the Cycling Canada Excel file.

**Choose CSV File**

Browse... bike.csv

Upload complete

**Select a league**

Elite

Elite  
Junior  
U23

**Select the last race to be included in analysis**

6-Sep-14

13-Apr-14  
27-Apr-14  
24-May-14  
31-May-14  
3-Aug-14  
10-Aug-14  
24-Aug-14  
6-Sep-14  
Run Bradley-Terry Ranking

The image shows a web interface for data filtering. It has three main sections. The first section, 'Choose CSV File', contains a 'Browse...' button and a text box with 'bike.csv'. Below this is a blue bar with the text 'Upload complete'. The second section, 'Select a league', features a dropdown menu with 'Elite' selected. The dropdown list shows 'Elite', 'Junior', and 'U23'. The third section, 'Select the last race to be included in analysis', has a dropdown menu with '6-Sep-14' selected. The dropdown list includes dates from '13-Apr-14' to '6-Sep-14', and a 'Run Bradley-Terry Ranking' option at the bottom.

Figure 4.6: Drop-down menus for choosing data filtering options.

**Choose CSV File**

Browse... bike.csv

Upload complete

**Select a league**

Elite

**Select a race level**

World Cup

**Select the first race to be included in analysis**

13-Apr-14

**Select the last race to be included in analysis**

24-Aug-14

Number of riders: 233  
Number of races: 7

**Enter weighting parameter (0 means unweighted)**

0

Run Bradley-Terry Ranking

**Data preview**

Show 25 entries

Search:

race_level	RACE_DATE	RACE_LOCATION	RIDER_CLASS	GENDER	RIDER	FINAL_RANK
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Esmeyer Martinus	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Skarnitzl Jan	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Scheiber Simon	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Batchelor Sebastian	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Jette Cameron	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Milatz Moritz	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Bird Tyron	
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Absalon Julien	1
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Fumic Manuel	2
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Marotte Maxime	3
World Cup	13-Apr-14	Pietermaritzburg, South Africa	Elite	Men	Cink Ondrej	4

Figure 4.7: Shiny user interface with data subset preview of the 2014 Elite World Cup season.

McConnell
×

Calculating paired wins... This might take a minute...

Daniel

Lindgren

Figure 4.8: Progress bar informing the user of algorithm details.



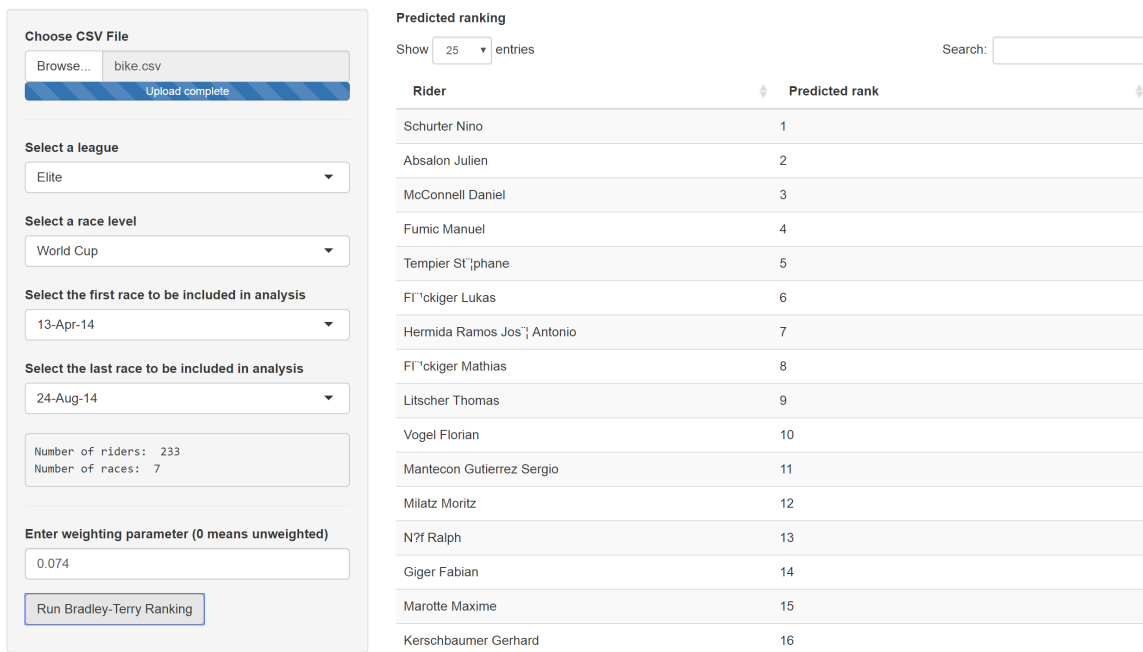


Figure 4.9: Example of a Bradley-Terry prediction based on all seven Elite World Cup races of the 2014 season with weighting parameter  $\theta = 0.074$ .

## Chapter 5

# Discussion

In our application of Bradley-Terry ranking for the Elite men’s UCI Mountain Bike World Cup race data, we improved on the flawed implementation by Anderson [1] and we proposed a weighted version of the classical Bradley-Terry model. Combining these statistical techniques with a graphical user interface, we created a successful example of a user-friendly yet computationally versatile interface with Shiny.

Through cross-validation, we estimated an optimal weighting parameter of  $\hat{\theta} = 0.074$ . Our predicted rankings performed better in terms of predictive error than the UCI point-based ranking. Furthermore, the MAE of the Bradley-Terry estimation decreased as we considered the top racers. To coaches and sports scientists, the performance of these top racers is often of higher importance than the rest of the field.

As shown in this project, the Bradley-Terry model merits further research in the context of prediction in racing sports. Discussed in Section 1.3, our probabilistic assumption is incorrect, but “not as incorrect” as previous literature. Work may be done to further refine the probabilistic assumption, while not sacrificing computational ease. It may also be beneficial to develop alternative weighting schemes and compare their performance with the exponentially weighted Bradley-Terry.

For cross-validation, we split our data based on 28 races and measured accuracy with MAE and RMSE. The cross-validation process may benefit from using data from more races or implementing some other model evaluation metrics. It would also be possible to perform validation on more races, perhaps the final three races. The R code for cross-validation is included in Appendix A and can serve as a starting point for future development.

Mentioned in Section 3.3, we used the variable  $t$  to suggest time. The races are not necessarily evenly spaced and therefore  $t$  is not a true time variable. Further work can be done to implement  $t$  as a true time variable such as the number of days prior to the most recent race. Its performance can then be compared with that of the current weighting scheme. It might be reasonable to instead consider a time subscript  $\alpha_{jt}$  on the ability score, but this greatly increases the parameter space and would make it difficult to obtain estimates.

Further, while CSIO researchers did not believe that individual race times were noteworthy for analysis, we believe that some form of race time analysis may produce interesting results. That is, racers who finish within a certain time difference  $\delta$  of another may be treated as having tied. The Bradley-Terry model can be extended to accommodate such ties [5], and further cross-validation can be performed to determine an optimal  $\delta$ .

Regarding the Shiny app, work can be done to increase its robustness in reading different data inputs. It would also be interesting if the user could choose a certain subset of riders they would like to be analysed. This would allow for predictions based on custom start lists and hypothetical scenarios. The R code for the Shiny app is included in Appendix B and can serve as a starting point for future development.

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# Appendix A

## Cross-validation code

```
1 options(useFancyQuotes = FALSE)
2 library(stringr)
3 library(hellno)
4 library(plotrix)
5
6 # Data directory
7 bike <- read.csv("C:/Users/Richard/Dropbox/Documents/SFU/Project/bike.csv",
8               header=TRUE,stringsAsFactors=FALSE)
9 # [1] "race_level" "RACE_DATE" "RACE_LOCATION" "RIDER_CLASS" "GENDER" "RIDER"
10 # [7] "FINAL_RANK" "COUNTRY" "UCI_CODE"
11
12 #####
13 # 1. DATA CLASSIFICATION #####
14 #####
15
16 ### Append RACE_YEAR onto dataset
17 data <- cbind(as.numeric(paste("20",str_sub(bike$RACE_DATE,-2),sep="")),bike)
18 colnames(data)[1] <- "RACE_YEAR"
19 years <- as.numeric(paste("20",unique(str_sub(data$RACE_DATE,-2)),sep=""))
20
21 ### Split data by RIDER_CLASS
22 elite <- data[data$RIDER_CLASS=="Elite",]; rownames(elite) <- NULL
23 junior <- data[data$RIDER_CLASS=="Junior",]; rownames(junior) <- NULL
24 U23 <- data[data$RIDER_CLASS=="U23",]; rownames(U23) <- NULL
25
26 ### Unique riders in each class
27 elite.riders <- unique(elite$RIDER)
28 junior.riders <- unique(junior$RIDER)
29 U23.riders <- unique(U23$RIDER)
30
31 ### Split each class by RACE_YEAR (automatic year detection)
32 for (i in years[1]:years[length(years)]){
33   elite.temp <- paste("elite.",i,sep="")
34   assign(elite.temp, data.frame(elite[elite$RACE_YEAR==i,],row.names=NULL))
35   junior.temp <- paste("junior.",i,sep="")
36   assign(junior.temp, data.frame(junior[junior$RACE_YEAR==i,],row.names=NULL))
37   U23.temp <- paste("U23.",i,sep="")
38   assign(U23.temp, data.frame(U23[U23$RACE_YEAR==i,],row.names=NULL))
39   print(c(elite.temp,junior.temp,U23.temp))
40 }
41
42 # Data by class and year
43 # [1] "elite.2011" "junior.2011" "U23.2011"
44 # [1] "elite.2012" "junior.2012" "U23.2012"
45 # [1] "elite.2013" "junior.2013" "U23.2013"
46 # [1] "elite.2014" "junior.2014" "U23.2014"
47
48 ### Race dates by class and year
49 for (i in years[1]:years[length(years)]){
50   elite.temp <- paste("elite.",i,".races",sep="")
51   assign(elite.temp, unique(elite[elite$RACE_YEAR==i,$RACE_DATE]))
52   elite.races <- unique(elite$RACE_DATE)
53   junior.temp <- paste("junior.",i,".races",sep="")
54   assign(junior.temp, unique(junior[junior$RACE_YEAR==i,$RACE_DATE]))
55   junior.races <- unique(junior$RACE_DATE)
56   U23.temp <- paste("U23.",i,".races",sep="")
57   assign(U23.temp, unique(U23[U23$RACE_YEAR==i,$RACE_DATE]))
58   U23.races <- unique(U23$RACE_DATE)
59   print(c(elite.temp,junior.temp,U23.temp))
60 }
61
62 # [1] "elite.2011.races" "junior.2011.races" "U23.2011.races"
63 # [1] "elite.2012.races" "junior.2012.races" "U23.2012.races"
```

```

64 # [1] "elite.2013.races" "junior.2013.races" "U23.2013.races"
65 # [1] "elite.2014.races" "junior.2014.races" "U23.2014.races"
66
67 ### Unique riders in each class and year
68 for (i in years[1]:years[length(years)]){
69   elite.temp <- paste("elite.",i,".riders",sep="")
70   assign(elite.temp, unique(as.data.frame(lapply(paste("elite.",i,sep=""),get))$RIDER))
71   junior.temp <- paste("junior.",i,".riders",sep="")
72   assign(junior.temp, unique(as.data.frame(lapply(paste("junior.",i,sep=""),get))$RIDER))
73   U23.temp <- paste("U23.",i,".riders",sep="")
74   assign(U23.temp, unique(as.data.frame(lapply(paste("U23.",i,sep=""),get))$RIDER))
75   print(c(elite.temp,junior.temp,U23.temp))
76 }
77
78 # Unique riders by class and year
79 # [1] "elite.2011.riders" "junior.2011.riders" "U23.2011.riders"
80 # [1] "elite.2012.riders" "junior.2012.riders" "U23.2012.riders"
81 # [1] "elite.2013.riders" "junior.2013.riders" "U23.2013.riders"
82 # [1] "elite.2014.riders" "junior.2014.riders" "U23.2014.riders"
83
84 #####
85 # 2. DATA FORMATTING #####
86 #####
87
88 ### Format all Elite races and riders into horizontal format
89 df.elite <- matrix(nrow=length(elite.riders),ncol=length(elite.races)+1)
90 df.elite[,1] <- elite.riders
91 df.elite <- as.data.frame(df.elite,stringsAsFactors=FALSE)
92
93 for (i in 1:length(elite.riders)){
94   for (j in 1:length(elite.races)){
95     if (length(elite[elite$RACE_DATE==elite.races[j]
96               & elite$RIDER == elite.riders[i],]$FINAL_RANK) > 0){
97       df.elite[i,j+1] <- elite[elite$RACE_DATE==elite.races[j]
98                             & elite$RIDER == elite.riders[i],]$FINAL_RANK
99     }
100   }
101 }
102
103 df.elite <- data.frame(cbind(data.frame(df.elite[,1]),
104                             data.frame(lapply(df.elite[,2:dim(df.elite)[2]], as.numeric))))
105 colnames(df.elite) <- c("Riders",elite.races)
106
107 ### Format all junior races and riders into horizontal format
108 df.junior <- matrix(nrow=length(junior.riders),ncol=length(junior.races)+1)
109 df.junior[,1] <- junior.riders
110 df.junior <- as.data.frame(df.junior,stringsAsFactors=FALSE)
111
112 for (i in 1:length(junior.riders)){
113   for (j in 1:length(junior.races)){
114     if (length(junior[junior$RACE_DATE==junior.races[j]
115                 & junior$RIDER == junior.riders[i],]$FINAL_RANK) > 0){
116       df.junior[i,j+1] <- junior[junior$RACE_DATE==junior.races[j]
117                                 & junior$RIDER == junior.riders[i],]$FINAL_RANK
118     }
119   }
120 }
121
122 df.junior <- data.frame(cbind(data.frame(df.junior[,1]),
123                               data.frame(lapply(df.junior[,2:dim(df.junior)[2]], as.numeric))))
124 colnames(df.junior) <- c("Riders",junior.races)
125
126 ### Format all U23 races and riders into horizontal format
127 df.U23 <- matrix(nrow=length(U23.riders),ncol=length(U23.races)+1)
128 df.U23[,1] <- U23.riders
129 df.U23 <- as.data.frame(df.U23,stringsAsFactors=FALSE)
130
131 for (i in 1:length(U23.riders)){
132   for (j in 1:length(U23.races)){
133     if (length(U23[U23$RACE_DATE==U23.races[j]
134               & U23$RIDER == U23.riders[i],]$FINAL_RANK) > 0){
135       df.U23[i,j+1] <- U23[U23$RACE_DATE==U23.races[j]
136                             & U23$RIDER == U23.riders[i],]$FINAL_RANK
137     }
138   }
139 }
140
141 df.U23 <- data.frame(cbind(data.frame(df.U23[,1]),
142                             data.frame(lapply(df.U23[,2:dim(df.U23)[2]], as.numeric))))
143 colnames(df.U23) <- c("Riders",U23.races)
144
145 ### Checksums
146 sum(df.elite[,2:(length(elite.races)+1)],na.rm=T) == sum(elite$FINAL_RANK,na.rm=T)
147 sum(df.junior[,2:(length(junior.races)+1)],na.rm=T) == sum(junior$FINAL_RANK,na.rm=T)
148 sum(df.U23[,2:(length(U23.races)+1)],na.rm=T) == sum(U23$FINAL_RANK,na.rm=T)
149
150 #####
151 # 3. CROSS-VALIDATION #####
152 #####
153
154 # 0. DATA FORMATTING #####

```

```

155
156 # options(warn=1)
157
158 raw <- rbind(elite)
159
160 data <- raw
161 riders <- unique(data$RIDER)
162 races <- unique(data$RACE_DATE)
163 data <- matrix(nrow=length(riders),ncol=length(races)+1)
164 data[,1] <- riders
165 data <- as.data.frame(data,stringsAsFactors=FALSE)
166
167 # Format race data horizontally
168 for (i in 1:length(riders)){
169   for (j in 1:length(races)){
170     if (length(raw[raw$RACE_DATE==races[j]
171               & raw$RIDER == riders[i,]$FINAL_RANK] > 0){
172       data[i,j+1] <- raw[raw$RACE_DATE==races[j]
173                       & raw$RIDER == riders[i,]$FINAL_RANK
174     ]
175   }
176 }
177
178 data <- data.frame(cbind(data.frame(data[,1]),
179                          data.frame(lapply(data[,2:dim(data)[2]],
180                                             as.numeric))))
181 colnames(data) <- c("Riders",races)
182 data[is.na(data)] <- 0 # Set all NAs as 0
183
184 ### 'data' is all races
185
186 # Checksum
187 sum(data[,2:(length(races)+1)],na.rm=T) == sum(elite[elite$RACE_YEAR <= 2014,]$FINAL_RANK,na.rm=T)
188
189 # Remove all world championship races
190 data <- cbind(data[,1:8], data[,10:17], data[,19:24], data[26:32])
191
192 # Subset test and validate sets
193
194 # race 27
195 Y_test <- data.frame(cbind(data[,1], as.numeric(data[, (dim(data)[2]-1)])),stringsAsFactors=F)
196 colnames(Y_test) <- c("Rider", "Actual")
197 order.Y_test <- order(as.numeric(Y_test$Actual), decreasing=F)
198 Y_test <- Y_test[order.Y_test,]
199 Y_test <- Y_test[Y_test$Actual!=0,]
200 rownames(Y_test) <- c()
201
202 # race 28
203 Y_val <- data.frame(cbind(data[,1], as.numeric(data[,dim(data)[2]])),stringsAsFactors=F)
204 colnames(Y_val) <- c("Rider", "Actual")
205 order.Y_val <- order(as.numeric(Y_val$Actual), decreasing=F)
206 Y_val <- Y_val[order.Y_val,]
207 Y_val <- Y_val[Y_val$Actual!=0,]
208 rownames(Y_val) <- c()
209
210 # race 1-26
211 X_test <- data
212 X_test <- X_test[,1:(dim(X_test)[2]-2)]
213
214 # Subset intersecting riders 1-26 and 27
215 X_test <- X_test[X_test$Riders %in% Y_test$Rider,]
216 rownames(X_test) <- c()
217 X_test <- X_test[which(rowSums(X_test[,2:dim(X_test)[2]])>0),]
218 rownames(X_test) <- c()
219
220 Y_test <- Y_test[Y_test$Rider %in% X_test$Riders,]
221 Y_test$Actual <- seq(1,dim(Y_test)[1])
222 rownames(Y_test) <- c()
223
224 test_riders <- length(unique(X_test$Riders))
225 test_pairs <- 0
226 test_races <- dim(X_test)[2]-1
227
228 # Calculate number of paired-comparisons 1-26
229 for (i in 1:test_races){
230   test_pairs <- test_pairs + sum(as.numeric(X_test[, (i+1)])!=0) - 1
231 }
232
233 # race 1-27
234 X_val <- data
235 X_val <- X_val[,1:(dim(X_val)[2]-1)]
236
237 # Subset intersecting riders 1-27 and 28
238 X_val <- X_val[X_val$Riders %in% Y_val$Rider,]
239 rownames(X_val) <- c()
240 X_val <- X_val[which(rowSums(X_val[,2:dim(X_val)[2]])>0),]
241 rownames(X_val) <- c()
242
243 Y_val <- Y_val[Y_val$Rider %in% X_val$Riders,]
244 Y_val$Actual <- seq(1,dim(Y_val)[1])
245 rownames(Y_val) <- c()

```

```

246
247 val_riders <- length(unique(X_val$Riders))
248 val_pairs <- 0
249 val_races <- dim(X_val)[2]-1
250
251 # Calculate number of paired-comparisons 1-27
252 for (i in 1:val_races){
253   val_pairs <- val_pairs + sum(as.numeric(X_val[, (i+1)] != 0) - 1
254 }
255
256 # 1. ESTIMATION #####
257
258 # Predicted ranking 1-30
259 pred <- c()
260 for (w in seq(0,0.1,0.001)){
261   # Calculate paired wins
262   X <- matrix(0, nrow=test_pairs, ncol=test_riders)
263   colnames(X) <- unique(X_test$Riders)
264   X_row <- 1
265
266   for (r in 1:test_races){
267     places <- max(as.numeric(X_test[, (r+1)]))
268     for (i in 1:(places-1)){
269       win <- which(X_test[, r+1]==i)
270       lose <- which(X_test[, r+1]==min(X_test[, r+1][X_test[, r+1]>i])) # next largest
271       if (length(win)>0 & length(lose)>0){
272         X[X_row, win] <- exp(-w*(test_races-r))
273         X[X_row, lose] <- -exp(-w*(test_races-r))
274         X_row <- X_row + 1
275       }
276     }
277   }
278
279   X[, dim(X)[2]] <- 0
280
281   Y <- rep(1, test_pairs)
282   model <- glm(Y~X-1, family=binomial(logit), control=glm.control(maxit=500))
283
284   rank <- data.frame(summary(model)$coefficients[,1])
285   rank <- data.frame(cbind(substr(rownames(rank), 2, 100), format(round(unname(rank), 2), nsmall=2)))
286   rownames(rank) <- c()
287   colnames(rank) <- c("Rider", "Estimate")
288   order.rank <- order(rank$Estimate, decreasing=TRUE)
289   rank <- rank[order.rank,]
290   #rank <- rank[rank$Rider %in% actual$Rider,] # remove riders from predicted not in actual
291   rownames(rank) <- c()
292
293   pred <- cbind(pred, as.character(rank[,1]))
294   print(w)
295 }
296
297 pred <- cbind(seq(1, dim(pred)[1]), pred)
298 pred <- data.frame(pred)
299 colnames(pred) <- c("Place", seq(0,0.1,0.001))
300
301 # Comparison of actual with predicted rankings
302 comp <- Y_test
303 for (i in 1:length(seq(0,0.1,0.001))){
304   pred.vec <- c()
305   for (j in 1:length(Y_test$Rider)){
306     if (length(pred[,1][pred[, (i+1)]==Y_test$Rider[j]])==0){
307       pred.rank <- NA
308       pred.vec <- c(pred.vec, pred.rank)
309     } else {
310       pred.rank <- pred[,1][pred[, (i+1)]==Y_test$Rider[j]]
311       pred.vec <- c(pred.vec, pred.rank)}
312   }
313   comp <- cbind(comp, pred.vec)
314 }
315 colnames(comp) <- c("Rider", "Actual", seq(0,0.1,0.001))
316 comp <- comp[complete.cases(comp),]
317 rownames(comp) <- c()
318 comp_test <- comp#[1:45,]
319
320 # Calculate MAEs
321 mae <- c()
322 rmse <- c()
323 for (i in 1:length(seq(0,0.1,0.001))){
324   err <- 1/dim(Y_test)[1] * sum(abs(as.numeric(as.character(comp_test[,2]))
325                                     -as.numeric(as.character(comp_test[, (i+2)]))))
326   mae <- c(mae, err)
327   err2 <- sqrt(1/dim(Y_test)[1] * sum((as.numeric(as.character(comp_test[,2]))
328                                     -as.numeric(as.character(comp_test[, (i+2)])))^2))
329   rmse <- c(rmse, err2)
330 }
331
332 par(mfrow=c(1,2), mar=c(4,4,2,1))
333 plot(seq(0,0.1,0.001), mae, col="red", ylab="MAE", xlab=expression(theta), cex=0.5, pch=20)
334 mae_lowess <- lowess(seq(0,0.1,0.001), mae, f=1/3)
335 lines(mae_lowess)
336 plot(seq(0,0.1,0.001), rmse, col="blue", ylab="RMSE", xlab=expression(theta), cex=0.5, pch=20)

```



```

337 rmse_lowess <- lowess(seq(0,0.1,0.001),rmse,f=1/3)
338 lines(rmse_lowess)
339
340 seq(0,0.1,0.001)[which(min(mae_lowess$y)==mae_lowess$y)]
341 seq(0,0.1,0.001)[which(min(rmse_lowess$y)==rmse_lowess$y)]
342 min(mae_lowess$y)
343 min(rmse_lowess$y)
344
345 # 2. VALIDATION #####
346
347 # Predicted ranking 1-27
348 pred <- c()
349 for (w in seq(0,0.1,0.001)){
350 # Calculate paired wins
351 X <- matrix(0, nrow=val_pairs, ncol=val_riders)
352 colnames(X) <- unique(X_val$Riders)
353 X_row <- 1
354
355 for (r in 1:val_races){
356 places <- max(as.numeric(X_val[, (r+1)]))
357 for (i in 1:(places-1)){
358 win <- which(X_val[,r+1]==i)
359 lose <- which(X_val[,r+1]==min(X_val[,r+1][X_val[,r+1]>i])) # next largest
360 if (length(win)>0 & length(lose)>0){
361 X[X_row, win] <- exp(-w*(val_races-r))
362 X[X_row, lose] <- -exp(-w*(val_races-r))
363 X_row <- X_row + 1
364 }
365 }
366 }
367
368 X[,dim(X)[2]] <- 0
369
370 Y <- rep(1,val_pairs)
371 model <- glm(Y~X-1, family=binomial(logit), control=glm.control(maxit=500))
372
373 rank <- data.frame(summary(model)$coefficients[,1])
374 rank <- data.frame(cbind(substr(rownames(rank),2,100), format(round(unname(rank),2),nsmall=2)))
375 rownames(rank) <- c()
376 colnames(rank) <- c("Rider", "Estimate")
377 order.rank <- order(rank$Estimate,decreasing=TRUE)
378 rank <- rank[order.rank,]
379 #rank <- rank[rank$Rider %in% actual$Rider,] # remove riders from predicted not in actual
380 rownames(rank) <- c()
381
382 pred <- cbind(pred,as.character(rank[,1]))
383 print(w)
384 }
385
386 pred <- cbind(seq(1,dim(pred)[1]),pred)
387 pred <- data.frame(pred)
388 colnames(pred) <- c("Place", seq(0,0.1,0.001))
389
390 # Comparison of actual with predicted rankings
391 comp <- Y_val
392 for (i in 1:length(seq(0,0.1,0.001))){
393 pred.vec <- c()
394 for (j in 1:length(Y_val$Rider)){
395 if (length(pred[,1][pred[, (i+1)]==Y_val$Rider[j]])==0){
396 pred.rank <- NA
397 pred.vec <- c(pred.vec, pred.rank)
398 } else {
399 pred.rank <- pred[,1][pred[, (i+1)]==Y_val$Rider[j]]
400 pred.vec <- c(pred.vec, pred.rank)}
401 }
402 comp <- cbind(comp, pred.vec)
403 }
404 colnames(comp) <- c("Rider", "Actual", seq(0,0.1,0.001))
405 comp <- comp[complete.cases(comp),]
406 rownames(comp) <- c()
407 comp_val <- comp#[1:10,]
408
409 # Calculate MAEs
410 mae <- c()
411 rmse <- c()
412 for (i in 1:length(seq(0,0.1,0.001))){
413 err <- 1/dim(comp_val)[1] * sum(abs(as.numeric(as.character(comp_val[,2]))
414 -as.numeric(as.character(comp_val[, (i+2)]))))))
415 mae <- c(mae, err)
416 err2 <- sqrt(1/dim(comp_val)[1] * sum((as.numeric(as.character(comp_val[,2]))
417 -as.numeric(as.character(comp_val[, (i+2)]))))^2))
418 rmse <- c(rmse, err2)
419 }
420
421 par(mfrow=c(2,1),mar=c(4,4,2,1))
422 plot(seq(0,0.1,0.001),mae,col="red",ylab="MAE",xlab=expression(theta),cex=0.5,pch=20)
423 lines(lowess(seq(0,0.1,0.001),mae,f=1/3))
424 plot(seq(0,0.1,0.001),rmse,col="blue",ylab="RMSE",xlab=expression(theta),cex=0.5,pch=20)
425 lines(lowess(seq(0,0.1,0.001),rmse,f=1/3))
426
427 seq(0,0.1,0.001)[which(min(mae)==mae)]

```

```
428 | seq(0,0.1,0.001)[which(min(rmse)==rmse)]
429 | min(mae)
430 | min(rmse)
431 |
432 | mae[length(seq(0,0.074,0.001))]
433 | rmse[length(seq(0,0.074,0.001))]
```

## Appendix B

# Shiny app code

```
1 options(useFancyQuotes = FALSE)
2 library(stringr)
3 library(shiny)
4
5 # Define UI for data upload app ----
6 ui <- fluidPage(
7   # App title ----
8   titlePanel("Bradley-Terry Ranking v2.2"),
9   # Sidebar layout with input and output definitions ----
10  sidebarLayout(
11    # Sidebar panel for inputs ----
12    sidebarPanel(
13      # Input: Select a file ----
14      fileInput("file1", "Choose CSV File",
15               accept = c("text/csv",
16                          "text/comma-separated-values,text/plain",
17                          ".csv")),
18
19      # Horizontal line ----
20      tags$hr(),
21      uiOutput("leaguebox"),
22      uiOutput("levelbox"),
23      uiOutput("startrace"),
24      uiOutput("endrace"),
25      verbatimTextOutput("dimensions"),
26      tags$hr(),
27      uiOutput("theta"),
28      uiOutput("runbutton"),
29      p(),
30      verbatimTextOutput(("process"))
31    ),
32    # Main panel for displaying outputs ----
33    mainPanel(
34      uiOutput("mainHeader"),
35      # Output: Data file ----
36      dataTableOutput("contents")
37    )
38  )
39
40 # Define server logic to read selected file ----
41 server <- function(input, output, session) {
42   df <- reactive({
43     data <- read.csv(input$file1$datapath, header = TRUE, stringsAsFactors = FALSE)
44     bike <- cbind(as.numeric(paste("20", str_sub(data$RACE_DATE, -2), sep="")), data)
45     colnames(bike)[1] <- "RACE_YEAR"
46     bike
47   })
48
49   output$mainHeader <- renderUI({
50     if((flag$x)!=1){
51       p(strong("Data preview"))
52     } else {
53       p(strong("Predicted ranking"))
54     }
55   })
56
57   flag <- reactiveValues(x=0, ranking=NULL)
58
59   output$contents <- renderDataTable({
60     if(is.null(input$file1)) {
61       return(NULL)
62     } else if (is.null(input$file1)==FALSE & flag$x!=1){
63       return(filtered()[,2:dim(filtered())[2]])
64     }
65   })
66 }
```

```

64   } else if (flag$x==1){
65     return(flag$ranking)
66   }
67 }
68
69 output$leaguebox <- renderUI({
70   if(is.null(input$file1)) return(NULL)
71   selectInput("league","Select a league",unique(df()$RIDER_CLASS),"pick one")
72 })
73
74 output$levelbox <- renderUI({
75   if(is.null(input$file1)) return(NULL)
76   selectInput("level","Select a race level",unique(df()$race_level),"pick one")
77 })
78
79 output$startrace <- renderUI({
80   if(is.null(input$file1)) return(NULL)
81   selectInput("first","Select the first race to be included in analysis",
82     unique(df()[df()$RIDER_CLASS==input$league & df()$race_level==input$level,]
83     $RACE_DATE),"pick one")
84 })
85
86 output$endrace <- renderUI({
87   if(is.null(input$file1)) return(NULL)
88   selectInput("last","Select the last race to be included in analysis",
89     unique(df()[df()$RIDER_CLASS==input$league & df()$race_level==input$level,]
90     $RACE_DATE),"pick one")
91 })
92
93 filtered <- reactive({
94   temp <- data.frame(matrix(0,ncol=dim(df())[2]),stringsAsFactors = FALSE)
95   colnames(temp) <- colnames(df())
96   for (i in match(input$first,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE))):
97     match(input$last,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE)){
98     temp <- rbind(temp, df()[df()$RIDER_CLASS==input$league
99     & df()$RACE_DATE==unique(df()[df()$RIDER_CLASS==input$league,]
100     $RACE_DATE)[i],])
101   }
102   temp <- temp[-1,]
103   temp <- temp[temp$race_level==input$level,]
104 })
105
106 riders <- reactive({
107   unique(filtered()$RIDER)
108 })
109
110 numRiders <- reactive({
111   length(riders())
112 })
113
114 races <- reactive({
115   unique(filtered()$RACE_DATE)
116 })
117
118 numRaces <- reactive ({
119   length(races())
120 })
121
122 output$dimensions <- renderPrint({
123   if(is.null(input$file1)) return(cat("Please select a .csv file to begin..."))
124   if(match(input$first,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE)) >
125     match(input$last,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE))){
126     return(cat("You have selected a start date after \nthe end date, please revise to continue..."))
127   }
128   cat("Number of riders: ", numRiders(), "\nNumber of races: ", numRaces())
129 })
130
131 output$theta <- renderUI({
132   if(is.null(input$file1)) return(NULL)
133   if(match(input$first,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE)) >
134     match(input$last,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE))){
135     return(NULL)
136   }
137   #if(is.null(input$runtime)) return(NULL)
138   numericInput("theta","Enter weighting parameter (0 means unweighted)",value=0,min=0,step=0.01)
139 })
140
141 output$runbutton <- renderUI({
142   if(is.null(input$file1)) return(NULL)
143   if(match(input$first,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE)) >
144     match(input$last,unique(df()[df()$RIDER_CLASS==input$league,]$RACE_DATE))){
145     return(NULL)
146   }
147   #if(is.null(input$runtime)) return(NULL)
148   actionButton("runbt","Run Bradley-Terry Ranking")
149 })
150
151 observeEvent(input$runbt, {
152   progress <- Progress$new(session, min=0, max=10)
153   on.exit(progress$close())
154 }

```

```

155 progress$set(message = "Initializing...",value=1)
156
157 df.run <- matrix(nrow=numRiders(),ncol=numRaces()+1)
158 df.run[,1] <- riders()
159 df.run <- as.data.frame(df.run,stringsAsFactors=FALSE)
160
161 progress$set(message = "Formatting...",value=2)
162
163 # Converting to horizontal format
164 for (i in 1:numRiders()){
165   for (j in 1:numRaces()){
166     if (length(filtered()[filtered()$RACE_DATE==races()[j]
167       & filtered()$RIDER == riders()[i,]$FINAL_RANK) > 0){
168       df.run[i,j+1] <- filtered()[filtered()$RACE_DATE==races()[j]
169         & filtered()$RIDER == riders()[i,]$FINAL_RANK
170     }
171   }
172 }
173 df.run <- data.frame(cbind(data.frame(df.run[,1]),
174   data.frame(lapply(df.run[,2:dim(df.run)[2]], as.numeric))))
175 colnames(df.run) <- c("Riders",races())
176 df.run[is.na(df.run)] <- 0
177
178 # Conversion checksum
179 checksum <- sum(df.run[,2:(numRaces()+1)],na.rm=T) == sum(filtered()$FINAL_RANK,na.rm=T)
180 if (checksum==TRUE){
181   progress$set(message = "Checksum passed...",value=3)
182 }
183 try (if (checksum == FALSE){
184   output$process <- renderPrint({
185     cat("Starting...\nFormatting...\nChecksum failed...\nreview data file or contact admin.")
186   })
187   stop()
188 })
189
190 # Subset test and validate sets
191 progress$set(message = "Calculating pairs...",
192   detail="This usually takes several seconds...",
193   value=4)
194
195 X_test <- df.run
196 test_riders <- length(unique(X_test$Riders))
197 test_pairs <- 0
198 test_races <- dim(X_test)[2]-1
199
200 # Calculate number of paired-comparisons
201 for (i in 1:test_races){
202   test_pairs <- test_pairs + sum(as.numeric(X_test[,i+1])!=0) - 1
203 }
204
205 # Begin B-T estimation
206 X <- matrix(0, nrow=test_pairs, ncol=test_riders)
207 colnames(X) <- unique(X_test$Riders)
208 X_row <- 1
209
210
211 # Creating data Matrix X
212 progress$set(message = "Calculating paired-comparison outcomes...",
213   detail="This might take a minute...",
214   value=5)
215
216 for (r in 1:test_races){
217   places <- max(as.numeric(X_test[,r+1]))
218   for (i in 1:(places-1)){
219     win <- which(X_test[,r+1]==i)
220     lose <- which(X_test[,r+1]==min(X_test[,r+1][X_test[,r+1]>i)) # next largest
221     if (length(win)>0 & length(lose)>0){
222       X[X_row, win] <- exp(-input$theta*(test_races-r))
223       X[X_row, lose] <- -exp(-input$theta*(test_races-r))
224       X_row <- X_row + 1
225     }
226   }
227 }
228
229 X[,dim(X)[2]] <- 0
230 # Calculate paired wins
231 progress$set(message = "Fitting model...",
232   detail="This takes the longest...",
233   value=6)
234
235 Y <- rep(1,test_pairs)
236 model <- glm(Y~X-1, family=binomial(logit), control=glm.control(maxit=500))
237
238 progress$set(message = "Extracting ability estimates...",
239   value=9)
240
241 rank <- data.frame(summary(model)$coefficients[,1])
242 rank <- data.frame(cbind(substr(rownames(rank),2,100), format(round(unname(rank),2),nsmall=2)))
243 rownames(rank) <- c()
244 colnames(rank) <- c("Rider", "Estimate")
245 order.rank <- order(rank$Estimate,decreasing=TRUE)

```

```
246 |     rank <- rank[order.rank,]
247 |     rownames(rank) <- c()
248 |     print(head(rank))
249 |     show <- data.frame(rank$Rider, rownames(rank))
250 |     colnames(show) <- c("Rider", "Predicted rank")
251 |     flag$ranking <- show
252 |     flag$x <- 1
253 |   })
254 | }
255 |
256 | # Create Shiny app ----
257 | shinyApp(ui, server)
```