Resolvable Designs With Large Blocks

J. P. Morgan, Virginia Tech

The combinatorial study of resolvability in block designs goes back at least as far as the well-known Kirkman's (1850) schoolgirl problem. The notion entered the statistical lexicon with Yates' work on square *lattice* designs (1936, 1940), though the term "resolvable design" was introduced by Bose (1942). Yates' lattice designs were extended to rectangular lattices by Harshbarger (1946, 1949). Williams (1975) and Patterson and Williams (1976) introduced a large family of resolvable designs they termed α-designs. Williams et al (1976) derived optimal resolvable designs with two replicates from BIBDs. Bailey, Monod, and Morgan (1995) proved strong optimality for the affine resolvable designs introduced by Bose (1942). Resolvable BIBDs have received significant attention in both the combinatorial and statistical literature. John et al (1999) discussed the practical need for resolvable designs with unequal block sizes; for example, about half of 245 experiments examined by Patterson and Hunter (1983) had unequal block sizes. However, the main families of resolvable designs in the statistical literature, all named above, all have equal block sizes.

This talk examines resolvable designs with two blocks per replicate from an optimality perspective, without requiring block sizes to be equal. For small to moderate replication, equalizing block (rather than treatment) concurrences for given block sizes is often, but not always, the best strategy. Sufficient conditions are established for various strong optimalities, and a detailed study of E-optimality offered, including a characterization of the E-optimal class of incidence structures. Optimal designs are found to correspond to balanced arrays and an affine-like generalization. Majorization arguments play a central role.